A STUDY OF HOW WATER QUALITY FACTORS
CAN BE INCORPORATED
INTO WATER SUPPLY ANALYSIS

U.S. Army Engineer
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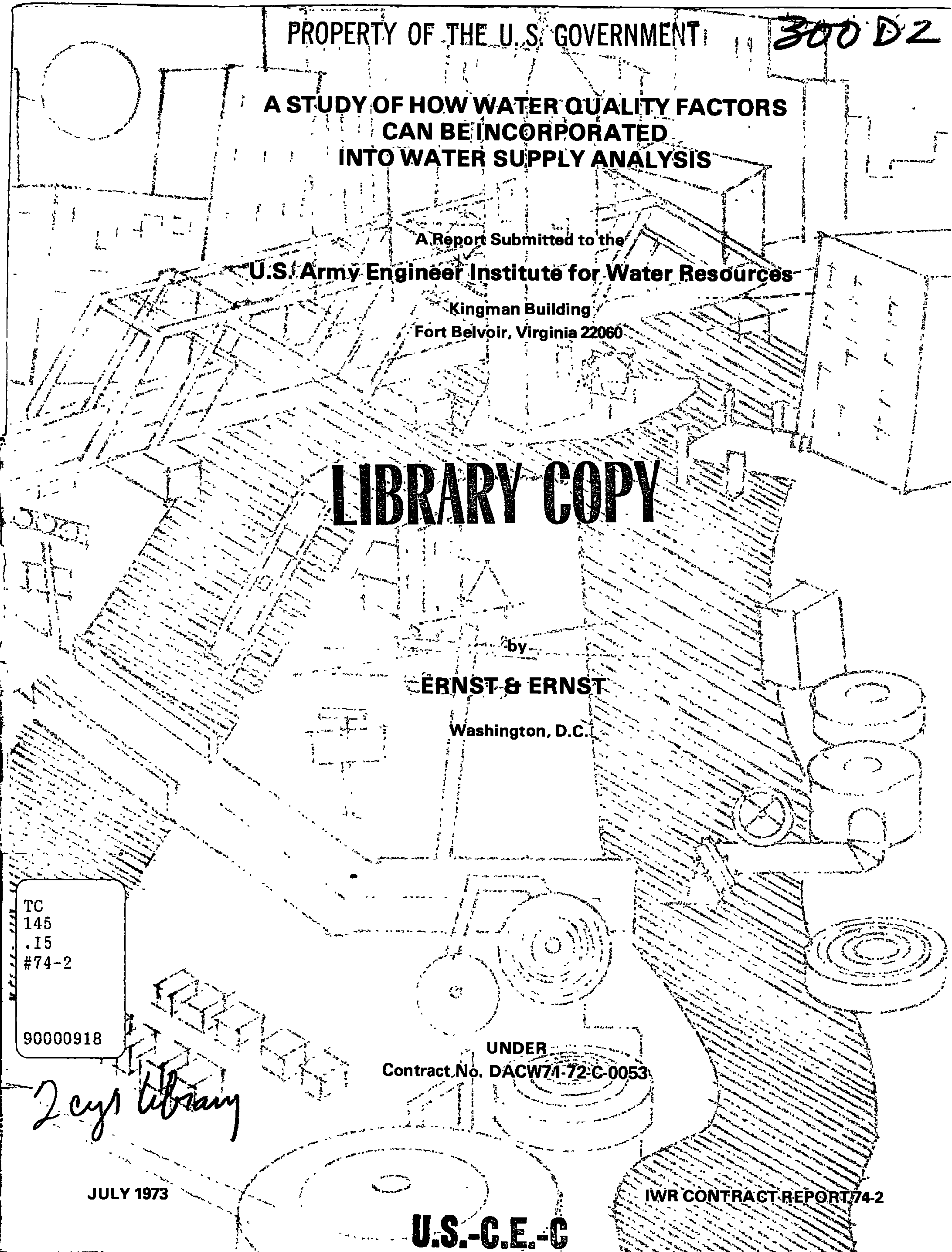

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A Report Submitted to the

U.S. Army Engineer Institute for Water Resources

Kingman Building
Fort Belvoir, Virginia 22060

by

ERNST & ERNST

Washington, D.C.

UNDER
Contract No. DACW71-72-C-0053

JULY 1973

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U.S.-C.E.-C

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(Three volumes bound in one)

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FOREWORD

This document constitutes the final report for "A Study of How Water Quality Factors Can Be Incorporated Into Water Supply Analysis," done by Ernst & Ernst under contract to the U.S. Army Corps of Engineers Institute for Water Resources. By means of both methodological and empirical economic analyses, the complete study examines supply and demand function concepts, as well as the notion of equilibrium water quantity, all within the context of how they are affected by changes in quality-related parameters.

The report is presented in a three-volume structure to facilitate ease of reading. As will be seen, Volume One serves as an overview of water supply functions. As such, it shows <u>why</u> it is feasible for a water treatment cost function to have a quality parameter embedded in it explicitly. Demonstrating (by example) <u>how</u> such a formulation can arise is accomplished in Volume Two. In Volume Three, the impacts of quality considerations on demand concepts are examined by means of examples with non-numerical functional forms. Pagination and reference lists/bibliographies are specific to each volume. In addition, the introductions to Volume Two and Three contain brief summaries of the preceding volume(s) as a means of ensuring continuity.

It should be noted at the outset that the purpose of this study was conceptual and methodological rather than empirical. Accordingly, the analytic derivations in Volumes Two and Three of cost and demand functions, respectively, are primarily intended to demonstrate derivational techniques. They are not intended to be precise "best" representations of the concepts under examination. For this reason, various formulations and functional forms used in these illustrative examples have been chosen for their mathematical manageability, not because they are advocated as the most comprehensive and realistic models one could construct. On the other hand, however, attention has been paid to empirical sense in each case so that intuitive properties are noted before any form is used.

The preceding points also apply to Volume Two's econometric estimations. There it will be shown that statistically-estimated coefficient values enable (and are meant for) the illustrative derivation of a cost function, but the econometric results per se are not the critical feature of that analysis. Rather, the statistical estimates merely serve to give numerical content to a function whose mathematically-tractable schematic form has already been specified.

Dr. Julian M. Greene was principal investigator and project director for the entire study. He conducted the research for the project and wrote the three volumes in the final report. Mr. Frederick L. McCoy contributed to the linear programming analyses, and Dr. William J. Leininger served as project supervisor.

VOLUME ONE

SUPPLY FUNCTION CONCEPTS, DEVELOPMENT, AND METHODOLOGY

VOLUME ONE

TABLE OF CONTENTS

I. STATEMENT OF THE PROBLEM

A. INTRODUCTION

This report is Volume One of a three-volume study prepared for the U.S. Army Corps of Engineers Institute for Water Resources (IWR) under Contract No. DACA 71-72-C-0053. This first volume concentrates on developing the conceptual framework and subsequent practical methodology for explicitly incorporating quality factors into the supply side of "water supply" analysis. Volume Two presents illustrative numerical examples of the techniques used in this volume. Volume Three examines the demand side of water supply analysis, again addressing the conceptual/methodological question of how quality parameters can be accounted for explicitly. The results of that development are then combined with Volume One's supply functions to obtain a notion of partial equilibrium ("supply = demand") water price-quantity combination that varies explicitly with quality.

A review of the literature revealed that published pragmatic methods for explicitly considering quality are very scarce. As the subsequent section points out, the predominant mode of operation appears to be to assume that all water is of the same quality, or simply to mention that quality may vary, but not to identify quantitatively how it varies or what impact these variations might have on supply. Hence, this effort breaks relatively new ground.

The rest of this chapter identifies the work statement for the whole project, the tasks and desired end products for Volume One, and the contents of the remainder of Volume One.

B. STUDY OBJECTIVES AND SCOPE

The problem statement and statement of objectives were concisely stipulated in the IWR research proposal. Quoting/paraphrasing that source is thus in order.

Problem Statement

> "Water resource planners and engineers are well aware of the fact that the public has in recent years become more and more sensitive to water quality. On the other hand, further quality improvement cannot be achieved without substantial increases in cost of pollution abatement measures and water treatment, and there are many instances where treatment is not effective. One possible solution would be to consider varying the standards for different uses so that the cost of making the additional improvement in quality is just equal to the incremental benefit derived from that particular use. If it were possible to differentiate the economic values associated with the various quality parameters, planners and engineers would be able to design alternative water supply and treatment systems which represent best allocation of resources."

Typical questions which planners and engineers could examine if the problem defined in this statement were successfully resolved include the following: Is there any economic gain in varying water quality standards for various uses? What is the cost of supplying water of varying standards? How can the quality factors be identified and systematically evaluated in water supply studies? Can the principle of marginal pricing be applied as the basis for cost sharing?

Objective of the Study

> "The objective of the study will be the development of a methodology which will make it possible to deal effectively with quality problems in studies of water supply. Most supply/demand studies assume a given 'quality' level and hence never deal with the issue when making estimates of demand or supply, or when considering resource allocation problems. Further, quality is seldom defined in terms of the factors which might reduce quality, i.e., reduce the supply available for particular uses. Such a methodology should enable Corps planners to give more adequate treatment to quality factors in water supply and to develop better plans consistent with optimal use of resources."

Based on this objective, a scope-of-study was ultimately formulated which delineates two phases covering the volume contents described earlier: Phase I (comprising Volume One) entails the development of a methodology for incorporating quality factors into water supply functions and resource allocation problems; while two-volume Phase II requires illustrative numerical examples of the Phase I techniques (Volume Two) and the conceptual development of analogous quality-inclusion methodology for water demand functions (Volume Three) alluded to before.

C. PHASE I TASKS AND OUTPUT

The tasks reported on in each volume are cataloged in their respective introductions. For Volume One, these are:

(1) Review the literature and documented experience to establish a limited number of classes of factors (pollutants) that impair quality.

(2) Review water quality requirements to determine types of demand characterized by the degree to which users can accept water quality impairment. In this connection, ascertain and review kinds of water quality indices that have been proposed.

(3) Develop methodology for reflecting quality in water supply functions for each class and level of water quality impairment factor. Consider segmenting supply by quality class and level and/or reflecting the cost of changing the class or level directly in an analysis.

(4) Convert the methodology into a pragmatic (technically correct) set of instructions as to how quality factors can be accounted for in regression or mathematical programming analyses.

Each of these tasks has been accomplished, as detailed in Chapters II-IV of this volume. In summary, the method for measuring the cost of varying standards for different users was explicitly developed from a classical economics optimization model. The derivation method for appropriate cost functions is described in Chapters II and IV where costs are shown to be an explicit function of a quality parameter which can represent any impairment. It is possible to set up pollutant classes based on common treatments required, particularly since treatment facilities are usually "package" operations.

With regard to the illustrative questions cited in the problem statement, the study results point toward a means of assessing economic gains if varying water quality standards are applied to different users by showing how to measure explicitly the cost of supplying water of different quality levels. Furthermore, the technique demonstrates that it is possible to deal directly with specific quality factors. Finally, the study clearly shows that a marginal cost concept makes sense in a water quality context; hence "marginal cost pricing" is feasible, _if_ a planner chooses to use it. It should be pointed out, however, that it is neither the intention nor purpose of this report to advocate marginal cost pricing as a recommended procedure. Indeed, it is not possible to do so here because it is a well-known fact of economic theory that knowledge of the relevant demand curve is critical for ascertaining if "marginal cost = price" is a sensible practice, and demand aspects are not considered in Phase I.

D. CONTENT OF THIS VOLUME

Chapter II develops the conceptual basis for incorporating quality in water supply functions. The key building blocks presented are the economic definition of supply, the concept of a quality indicator function, and the method for interrelating these two concepts to arrive at a quality-constrained derivation of a cost function. The chapter concludes with a discussion of the data needed to implement an analysis: water quality data for the index, abatement (treatment) costs, and the specified production function representing the treatment process.

Chapter III provides a full elaboration on the data needs specified in Chapter II. The major contribution of this chapter is the presentation of different quality indexes and the documentation of available sources of treatment cost data.

Chapter IV uses the concepts developed in Chapter II and the data format set forth in Chapter III to show how quality factors can be made explicit in supply analyses, and how these factors will impact on a derived treatment cost function.

Chapter V provides a reference list and bibliography of the rather extensive literature that was reviewed; it also summarizes the substance of contact meetings which took place.

II. CONCEPTUAL BASIS FOR INCORPORATING QUALITY INTO WATER SUPPLY FUNCTIONS

A. ECONOMIC DEFINITION OF SUPPLY

Most succinctly described, a supply curve provides a seller's answer to the question, "Without knowing what the relevant market price really is, what quantity would you be willing to offer for sale at each of the following prices...?"1/ From even this simple definition, however, it is clear that consideration of influencing factors other than price ("ceteris paribus" conditions, in economic jargon) is critical to a full understanding of supply. For example, before a seller can (would be willing to) say how much of an item he would offer for sale at a given price, he certainly would examine his materials and inputs costs. Only if the given price is favorable in relation to all such production costs will he be inclined to sell. Because of this, costs are an important determinant of supply, particularly as they appear in the form of marginal (incremental) costs. In fact, basic economic theory states that a profit-maximizing producer in a perfectly competitive market structure2/ takes his marginal cost curve _to be_ his supply curve as long as unit price exceeds his average variable costs. That is, facing a posted price, output (quantity supplied)

1/ Thus, supply is independent of demand, and it is precisely this independence that enables interaction and the logically consistent "quantity demanded = quantity supplied" determination of conceptual market equilibrium price.

2/ The main distinguishing characteristic of perfect competition is that sellers are price-takers, i.e., no one seller can affect market price, meaning each is presumed to face a horizontal demand curve.

should be set where "price = marginal cost" because a unit change (up or down) from that level reduces profit (this implies rising marginal costs, so "second order" optimization conditions are met). When one traces the locus of these "price = marginal cost" optima, the almost trivial result is merely a reproduction of the marginal cost schedule, which thus establishes it as indeed the indicator of quantities offered for sale at different prices; in other words, the competitive producer's supply curve.

In cases (more realistic and applicable for the current study) of <u>im</u>perfect competition, where a seller is presumed able to affect the price he receives at least to some extent, the above analysis must be modified, but the importance of marginal costs remains.[3/] Without presenting all the attendant economic theory background, suffice it to say that here, where the demand curve facing a seller is a "price vs. output" function of output and not horizontal, there is not a single "given price." Thus, the seller's incremental (marginal) revenue MR is a schedule of values (generally less than price) derived from the demand curve and corresponding to different output levels. When this seller determines his optimal output, he employs the "marginal" principle described previously, but now he equates marginal cost (MC) to MR instead of to price and then ascertains what price to charge by reference to his demand curve. In short, given a static demand curve (which, by definition, depicts a range of different price and quantity combinations), there will be only one price and quantity combination desired by the imperfectly competitive seller. It is not possible to "face" him with different prices and record his quantity-supplied response to each;

3/ For a good discussion of the monopolists' supply curve, see C. E. Furguson (11), pp. 300-302. (The non-horizontal demand curve facing the seller is taken to represent all forms of imperfect competition since, in general, such is the case with oligopoly, monopolistic competition and monopoly.) Throughout this report the parentheses source citations refer to numbered bibliography listings.

his "supply curve" consists of a single point in the price vs. quantity space.[4/]

In summary, we have seen that even though a supply curve, *per se*, is not derivable for the seller in imperfect competition, reference to his marginal costs is still critical to his determination of what quantity to supply, as it was unquestionably in the perfectly competitive seller case. Generalizing, when one considers the question of a supply function, he cannot do so without simultaneously examining marginal costs. Such is indeed the basis for much of the analysis which follows.

B. NOTION OF A QUALITY INDICATOR FUNCTION

Achieving different quality levels can involve varying production techniques, for example, with a resultant effect on quantity supplied of an item. It can be argued that quality differences in fact define separate entities, each of which would have its own supply function. There are undoubtedly cases in which quality differences accompany physical differences significant enough to define virtually distinct things so that individual supply functions can be analyzed on the basis of the physical characteristics alone, without direct consideration of quality factors. For water supply, however, quality is perhaps *the* main distinguishing trait (among different uses, as will be seen); consequently, it must be dealt with explicitly. How to do this, of course, is the central purpose of this entire report.

Quality will be explicitly dealt with through the use of a "quality indicator function" (QIF). The QIF can take either of two forms:

(1) Direct measures of individual quality factors and corresponding tolerances permitted for each

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4/ Ferguson, *ibid*, shows that a prescribed set of demand curve shifts will enable deriving a price-output response locus that is specific to the stipulated shifts and thus constitutes a quasi-supply curve.

(2) A summary function which translates observations such as in (1) into alternative usable arrangements.

Form (1) refers to the actual process of monitoring impairment factors such as turbidity, alkalinity, suspended solids, etc.; comparing the data to suggested "norms"; and then fashioning a quantitative gauge of how closely the observed values conform to the recommended levels for each impairment. One means of doing this (presented in detail as an illustrative example in Section IV-A of this report) is to derive a supply function via a constrained optimization process that has accounted for a stipulated impairment tolerance as one of the constraints. This builds into the function itself an allowance for the prescribed tolerance and enables a judgment of how varying the tolerance level would affect supply.

Most succinctly stated, Form (2) takes the appearance of an index number. Unlike Form (1), this kind of QIF permits including several quality factors into a single dimensionless number. As will be seen in Section III-A, computation of indices can be accomplished by recording actual observations, soliciting knowledgeable persons' opinions about how "good" or "bad" a body of water is, or using a combination of both methods.

It is clear that, in the context of usable supply, both forms of QIF attempt to evaluate characteristics of treated water. Preparatory to the theoretical analysis of the next section, therefore, let q denote, say, gallons per day of treated water flowing from a treatment plant. It then follows that the QIF can be written symbolically as $I = h(q)$, where the function $h(q)$ represents a sampling or testing function applied to the flow of treated water q, and I is the numerical result of the testing process (the quality index). This schematic formulation thus covers both Form (1) and (2) QIF's.

C. QUALITY - CONSTRAINED COST MINIMIZATION SCHEMATIC DERIVATION OF A SUPPLY FUNCTION

In Section II-A, it was implied that one interpretation of a supply curve is that it depicts the minimum unit price at which any specified incremental quantity of an entity will be offered for sale. That is, a supplier would be glad to offer the same specified incremental quantity at any higher price, but since he would be unwilling to accept a price below his lowest additional cost for producing/providing this additional quantity, he takes the locus of these minimum (incremental cost) points as his effective supply "guide." This interpretation provides the rationale for deriving each effective supply point by first minimizing the total cost of providing each possible output level and then obtaining the marginal cost function that corresponds to the derived total cost function.[5/] Extension of this traditional "classical" economic analysis will show how water quality allowances can be incorporated into supply analysis.

Consider, therefore, a water treatment plant that processes "contaminated" raw water, refining it to "clean" usable water. Let:

x_1 = gallons of raw water per day

x_2 = amount of "composite" treatment input used per day. This variable has a purposely broad connotation, being in fact a generic term. Thus x_2 can represent any or all of treatment activities such as chlorination, by-pass piping,

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5/ For description, including mathematical analysis, of constrained cost minimization, see J. Henderson and R. Quandt (13), pp. 52-79. Note also at this point that allegations about specific production characteristics for water treatment (e.g., that it is a discontinuous, rather than continuous, process [the former claim not supported by information in Chapter III]) are largely moot in a conceptual development such as this study. The point to make is that, although details of how to compute it may need clarification, the concept of marginal cost is well-defined and applicable here; consequently, presenting a general model to depict the essential economic theory and reasoning is both efficient and most easily understood. Reiterating, "supply curve" and "marginal cost" are being used "synonymously" because the latter is important in deriving the former for all market structures. Thus, this text discussion is not reverting to perfect competition; it merely demonstrates the marginal cost function derivation process for any seller type.

aeration, low-flow augmentation, filtration, etc., that it needs to in order to characterize a system or process. For the illustrative examples presented in this study, x_2 will refer to specific treatments designed to correct specific impairments, but this is not meant to limit the general interpretation it is intended to be capable of assuming. (In this connection, reference is made to Chapter IV where this point is discussed in greater detail with respect to data requirements.)

w_1, w_2 = respective unit input prices

F = (investment) cost of plant

r = annual capital charge rate on plant

q = gallons of treated water output per day, as in Section II-B

$g(x_1, x_2, F)$ = "production function" which characterizes the treatment plant and reflects the technological process of transforming x_1, x_2, and F into q.

I = h(q) is Section II-B's QIF, "quality" being a function explicitly of q and implicitly of x_1, x_2, and F. It is intended that h(q) represents either a sampling/testing function applied to q, or an implicit functional relation among x_1, x_2, and F that characterizes an impairment level constraint. (One form of the latter will be presented later, in Section IV-A.)

It is now desired to minimize daily costs (with respect to the three inputs) of producing a specified amount of treated water (q*) that is of a specified quality (I*). Thus, the task is to minimize (with respect to x_1, x_2 and F)[6/]

$$C = w_1 x_1 + w_2 x_2 + rF \tag{1}$$

subject to

$$q^* = g(x_1, x_2, F) \quad \text{(the output-production function constraint)} \tag{2}$$

and

$$I^* = h(q) = h(g[x_1, x_2, F]) \quad \text{(the quality constraint).} \tag{3}$$

6/ A somewhat similar, although much more abstract, model intended for different purposes is given by J. H. Boyd in Kneese and Bower (18), p. 207. Recall that the intent here is to present the theoretical rationale for quality parameter inclusion--hence, the schematic formulation. Illustrative actual function derivation is discussed in Volume Two.

Assuming that h( ) and g( ) have continuous second partial derivatives, then the above problem is well-defined and directly amenable to the standard Lagrange multiplier constrained optimization tool of economic analysis. With λ and θ as the to-be-determined multipliers, the relevant Lagrangian function is

$$L = w_1 x_1 + w_2 x_2 + rF + \lambda[q^* - g(x_1, x_2, F)] + \theta\Big[I^* - h(g[x_1, x_2, F])\Big]. \quad (4)$$

Denote the partial derivatives of g with respect to x_1, x_2, and F by, respectively, g_1, g_2, and g_F; and let h' be the derivative of h with respect to q. Then the familiar first-order optimization equations that result from setting first partials of L (with respect to x_1, x_2, F, λ and θ) equal to zero can be written compactly as

$$w_i - \lambda g_i - \theta h' g_i = 0 \quad \text{(two equations, as } i = 1, 2) \qquad (5)$$

$$r - \lambda g_F - \theta h' g_F = 0 \qquad (6)$$

Equation (2) (7)

Equation (3). (8)

These five equations ((5) - (8)) are conceptually solvable for the five unknowns x_1, x_2, F, λ and θ as functions of the known parameters in the model, namely w_1, w_2, r, and, of special importance, q^* and I^*. (In symbolic form, for example, $x_1 = x_1(w_1, w_2, r, q^*, I^*)$ is the solution for x_1.) Inserting the x_1, x_2 and F solutions into the Cost Equation (1), costs can be stated as a function of input prices, output q^* and quality level I^*. Since q^* was taken as <u>any</u> given output, its "*" superscript can be dropped, leaving the just-derived function as the locus of minimimum costs of producing any level of output flow q; in short, one has derived the standard (long-run) cost function of economic analysis, but with the added feature that the quality index appears explicitly:

$C_{LR} = \varphi$ (prices; q; I*). (9)

Finally, the marginal cost function, shown previously as critical to determining the plant's supply, is derived by simply differentiating C_{LR} partially with respect to q (subscript notation again signifying partial derivative):

$MC = \varphi_q$ (prices; q; I*). (10)

With marginal costs an explicit function of I*, it can thus be concluded that there is indeed sound conceptual basis for incorporating quality considerations directly into water supply functions. Moreover, this analysis achieves added importance because it demonstrates a potentially empirically operational method for actually deriving quality—specific supply. This conclusion is expanded upon in Section II-D.

In concluding this discussion, it is useful to present results derived here in traditional graphical terms. Accordingly, Figure II-1's \$ vs. q (panel a) and \$/q vs. q (panel b) graphs give, not the familiar single curve presentation, but rather families of curves, indexed by I* values. Figure II-1 depicts the hypothetical case of decreasing marginal costs (to reflect the increasing returns-to-scale, or economies-of-scale, generally alleged to characterize capital-intensive water treatment "production" processes) and the presumption that total and incremental costs of high quality I_2^* exceed those of low quality I_1^* at each feasible level of output. Thus, a plant producing q_0 gallons of treated water each day realizes total costs of, respectively, C_1 and C_2, depending on if Quality 1 or 2 is achieved, and corresponding marginal costs MC_1 and MC_2. The difference ($C_2 - C_1$) is therefore the incremental cost of achieving a quality increase of amount ($I_2^* - I_1^*$) at an operating level of q_0 gallons per day. Alternatively stated, ($C_2 - C_1$) measures the "cost" of ($I_2^* - I_1^*$) amount of additional pollution, measured at a q_0 feasible treatment rate.

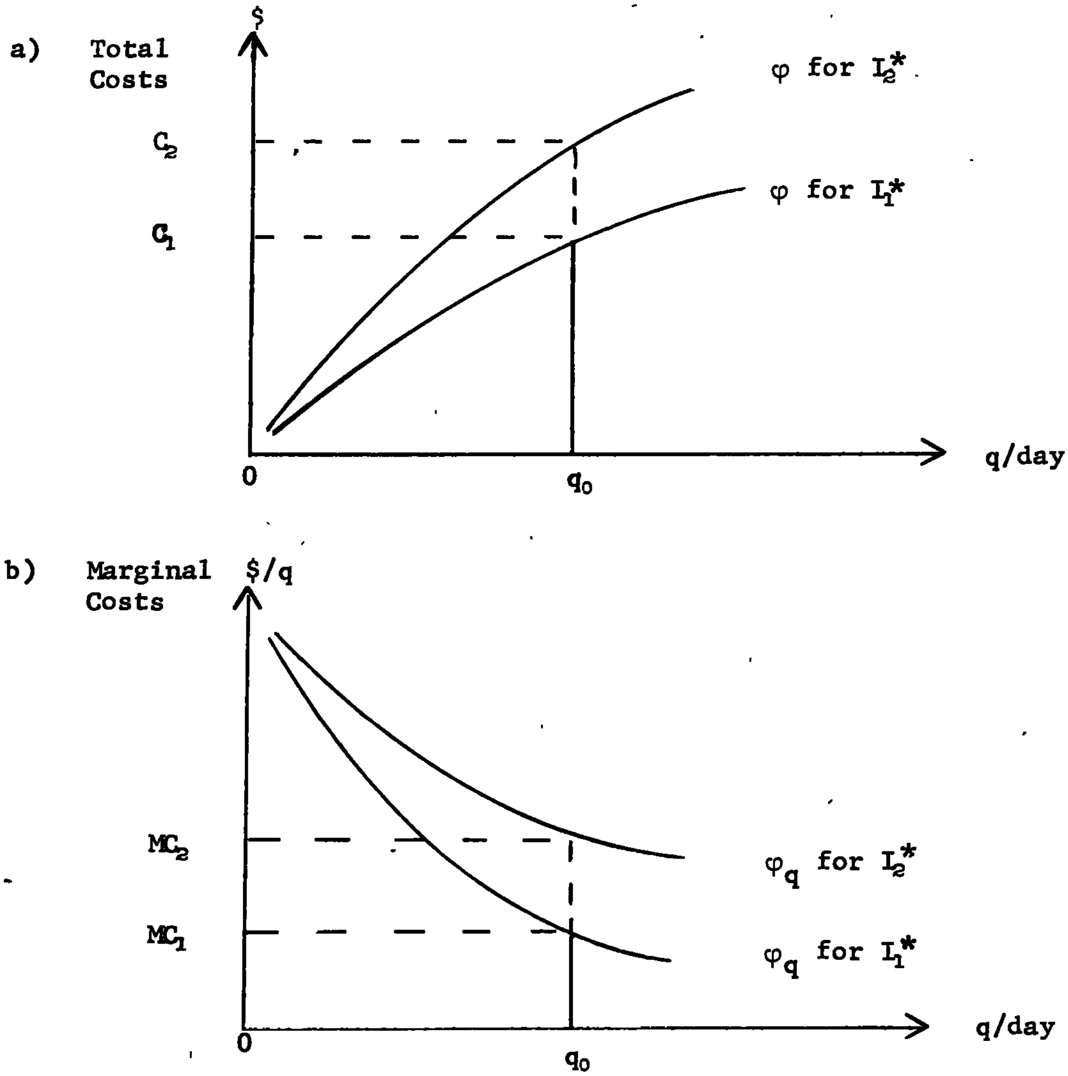
FIGURE II-1

HYPOTHETICAL TOTAL AND MARGINAL COST FUNCTIONS
FOR HIGH (I_2^*) AND LOW (I_1^*) QUALITY WATER

Total Costs $C_{LR} = \varphi$ (prices; q; I*)

Marginal Costs MC = φ_q (prices; q; I*)

$I_2^* > I_1^*$



D. FACTORS REQUISITE FOR AN EMPIRICALLY OPERATIONAL ANALYSIS

It is clear that the preceding development can be applied, provided three data elements are available: observations on quality impairment factors that permit Equation (3)'s QIF constraint to be quantified; input price data (values of w_1, w_2, and r); and a specified production function which represents the treatment plant's "productive process" (in other words, numerical specification for Equation (2)'s output-production function constraint). The analysis has shown that each of these pieces of information would be critical to the actual derivation of a cost function embodying quality parameters. It is the purpose of Chapter III to review in some detail what observations can be obtained for each of these categories. Volume Two will present an illustrative example of how quality influences supply considerations by applying Equations (2) and (3) to a data base and functional forms in order to derive an actual cost function.

III. DEVELOPMENT OF THE COMPONENTS FOR IMPLEMENTING THE ANALYSIS

A. WATER QUALITY ASPECTS

Recall that the QIF (the h(q) function) was made general so as to represent either of the two forms it would be likely to assume, Form (1)'s formulation using data on individual impairments, or Form (2)'s summary index. Common to both of these forms, however, is certain fundamental information regarding water quality questions. The primary purpose of this section is to catalog these data, indicating their relevance to the model developed in Chapter II.

Classes of Water Use

A natural starting point for any study of water is to recognize that different uses of water require different quality characteristics (for example, dissolved solids concentrations of 35,000 milligrams per liter (mg/l) are permissible in some industrial uses, whereas a maximum of 500 mg/l is recommended for public water supply).[7/] Because of this, it is tempting to segment a supply analysis on the basis of water use. It is not necessary to do so and, furthermore, such an approach could conceivably mask somewhat the central issue of how quality parameters actually affect supply functions. The masking would come about by creating a separate function for each quality

7/ From National Technical Advisory Committee, henceforth denoted NTAC (26), pp. 20 and 189. The subsection subsequent to this reviews users' quality criteria more extensively.

level so that comparisons of functions would be required in order to gauge quality effects instead of direct determination by reference to one function.

Thus, user groups served in the present study as an efficient means by which to determine precisely which quality characteristics must be dealt with. Accordingly, there proved to be ample support in the literature[8/] for the following eight categories as a comprehensive listing of water uses:

- Public drinking supply
- Aquatic (fish/shellfish) and wildlife propagation
- Recreation and aesthetics
- Industrial supply
- Agricultural use
- Water power (mainly hydroelectric)
- Navigation
- Disposal of industrial wastes and sewage.

Finer breakdowns are possible for agricultural use (into irrigation, livestock, and home groups), and these have ultimately been included. Similarly, "industrial supply" can be subdivided by specific industries, but it was questionable that doing so would add appreciably to the results of what is primarily a methodological analysis. The other headings can stand as they are and, in general, are self-explanatory.[9/]

Impairment Limits by Use Classes

Probably the most comprehensive analysis of water impairment tolerances by user is found in NTAC's (24) Water Quality Criteria. It succeeded

8/ See, for example, NTAC (26), Secs. I-V; Kneese and Bower (18), p. 6; McKee and Wolf (22), pp. 88-122; and T. R. Camp (4), p. 209.

9/ Possible exceptions are aquatic and wildlife propagation (which refers to water used as natural habitat for animal species) and waste disposal (refering to the natural assimilative capacity of water courses).

and up-dated McKee and Wolf (20), this latter being a pioneering work in breadth of coverage. Of note too is the fact that with respect to drinking water criteria, NTAC has included the most recent (1962) revisions of the U.S. Public Health Service recommended standards as found in U.S. Department of Health, Education, and Welfare-USPHS (28). Furthermore, drinking water criteria proposed by the United Nations World Health Organization (WHO) and the American Water Works Association (AWWA) are virtually comparable to NTAC.[10/] In short, NTAC is sufficiently complete and representative to stand alone as a source of water quality tolerance.

It is immediately apparent upon examination of this document that the list of potential impairments is rather extensive, numbering 50 items for drinking water alone (see NTAC (26), p. 20). Included among these, however, are 37 organic (e.g., pesticides), inorganic (certain trace elements), and radiological substances that are little affected by common water treatment processes (implying that if any such item were to be present in concentration exceeding recommended levels, special treatment would be required). This means that specific information on treatments (and corresponding costs thereof) to deal with such items is not readily available, in addition to which the scientific knowledge required to cope with the chemical properties is beyond the scope of this study. For both the reasons indicated and because there is no real loss of generality as far as application methodology is concerned, the list of impairments ultimately surveyed consisted of the following physical, microbiological, and inorganic items:

10/ Page 20 in NTAC (26) and page 1685 in Linstedt, et al. (20) afford comparison of WHO and AWWA criteria with NTAC.

- <u>Physical</u>
  - Taste
  - Color
  - Odor
  - Temperature

- <u>Microbiological</u>
  - Bacteria Count (Bacteria, especially fecal coliforms, have been used as indicators of the sanitary quality of water for nearly a century.)

- <u>Inorganic Chemicals and Related Items</u>
  - BOD (Biochemical oxygen demand is a measure of how heavily taxed the natural assimilative capacity of a watercourse is. BOD varies inversely with DO [dissolved oxygen content of a waterway], often listed as a separate quality factor. Because treatment processes seem to cite BOD, not DO, as primarily affected, it is the parameter considered here.)
  - Suspended Solids (items carried by, but not in solution with, a water course)
  - Total Dissolved Solids (mainly elements and salts in solution)
  - pH (a measure of dissociated hydrogen ions in a liquid and thus an indicator of acidity/alkalinity)
  - Hardness (generally a measure of the concentration of $CaCO_3$ [calcium carbonate] in water, such being associated with "scaling" on, e.g., pipes).

The "operational adequacy" of this list is further attested to by similar lists in Hall and Dracup (12, p. 31); Kneese and Bower (18, p. 14); Fair, <u>et al.</u> (10, p. 294) to a slightly lesser extent; and Clark, <u>et al.</u> (5, pp. 240-241).

In summary, then, data from NTAC were used to construct worksheet arrays of impairment tolerances by user for the first five user classes listed in the previous subsection. Such information was drawm from the "summary" or "discussion" headings of NTAC Sections I-V. McKee and Wolf (22, p. 122) provided the general ("Limiting...concentrations...of pollutants are seldom, if ever, found in the literature") guides for water power and navigation uses. Criteria for waste disposal use were developed from an assimilation of the literature surveyed.

Impairment Treatment Processes

In an attempt toward determining costs of alleviating various water pollutants (and thereby deducing the cost of different quality levels), a search of (primarily engineering) literature was first conducted to ascertain what physical/biological/chemical treatments are most generally used to remove or reduce the specific contaminants delineated in the previous sub-section. Three preliminary general observations emerged from this effort:

(1) Water treatment facilities often come as "packages" so that it may be difficult to associate a specific piece of equipment or process step with a specific impairment. Thus, one impairment may require multiple treatment steps. This fact is particularly critical as regards trying to compute empirically the cost of treating one certain pollutant.

(2) The counterpart to (1) is the fact that as a rule a treatment is not specific to one contaminant but rather will affect two or more (e.g., rapid sand filtration treats both bacteria and turbidity). The "costing out specifics" comments in (1) again apply.

(3) Although the end results quite likely differ, the types of steps/processes used by, respectively, wastewater treatment (treating outflow so as to produce higher quality effluent, regardless of intended use of the effluent) and water purification (the treating of inflow so as to produce higher quality acceptable for immediate use) are, in general, the same. (However, specific types of equipment may differ somewhat as, for example, stronger comminutors may be needed for the latter than for the former because of coarser floating material encountered.) The point is mentioned here because in the literature one

usually reads about either wastewater treatment or water purification but not both at the same time, whereas, in this study both can rightfully comprise part of the "water supply" process.[11/] Furthermore, even if cost and specific equipment differences exist, the analytic approaches to both procedures are, for the purpose of this conceptual study, virtually identical.

Despite these observations, and, in addition, the rather technical nature of the engineering sources, it was possible to determine what impariment a treatment is primarily meant to correct. This information is presented in Table III-1, the main sources for which were Culp and Culp (7, p. 271ff); Johnson (17, pp. 26-36); AWWA (1, ch. 17); Clark, et al. (5); Fair, et al. (10, ch. 11); and McGauhey (21, pp. 266-276).

Use Classes Grouped by Impairment Treatment

Table III-1 ("treatments vs. impairments") can now be combined with the "impairments vs. users" information previously discussed by considering what impairments most critically need alleviating for a particular use and then listing the treatments specific to those quality factors. The result of such combination is the "treatment vs. user" tableau, Table III-2, whose entries do, in fact, note what impairments, specific to a use (column heading), are relieved by an indicated treatment (row heading).

Further explanation of how to read Table III-2 is in order. As an illustration, the tabular entry in Column 2 (public water supplies) across from

11/ To expand on this point, "wastewater treatment" steps are usually identified sequentially as preliminary, primary, secondary, and tertiary. With respect to Table III-1, these steps correspond roughly to the first two, the third and fourth, the fifth through tenth, and the last two treatments, respectively, listed there. "Water purification" is generally considered to consist of, at least, coagulation, sedimentation, filtration, and chlorination. Thus, there is clear similarity.

TABLE III-1

COMMONLY-USED WATER TREATMENTS AND PRINCIPAL IMPAIRMENTS AFFECTED

Treatment	How It Works	Main Impairment(s) Corrected
Screening	Water flows through porous grates	Large floating items
Grit Chambers Comminutors	Shredding and separating devices	Gross solid pollutants
Coagulation, Flocculation	Chemicals (alum, hydrated lime) cause agglomeration on surface waters	Turbidity, color, taste, odor, phosphorus, pH (coagulant aid especially effective)
Sedimentation	Removal of solid particles by gravity settling	Suspended solids, turbidity, some BOD effect
Slow Sand Filter (not often used)	Separates substances by combination of straining, adsorption, flocculation	Bacteria, turbidity, color
Rapid Sand Filter	Sand/gravel medium removes non-settleable floc and impurities remaining from coagulation	Bacteria, turbidity
Trickling Filter	Waste effluent is sprayed over rock bed on which micro-organisms grow and feed on organic matter	BOD, suspended solids, bacteria
Activated Sludge	Process of aerating waste water so microbiological waste metabolism will be faster	BOD, suspended solids, bacteria
Stabilization	Çachement for impounding water until organic wastes stabilize and aerobic decomposition occurs	BOD, suspended solids, bacteria
Disinfection (mainly chlorine)	Hydrochlorous acid forms, enabling chlorine to destroy bacteria cell's enzymatic processes	Bacteria (to lesser extent: odor, corrosion, BOD)

TABLE III-1 (Cont.)

Treatment	How It Works	Main Impairment(s) Corrected
Activated Carbon	Removes organic contaminants by adsorption	Taste, ordors
Ion Exchange	Exchange resin removes certain metal ions in exchange for sodium	Hardness, dissolved solids, pH adjustment; also chlorine removal (Note: reverse osmosis, electrodialysis, and distillation also affect these parameters)

TABLE III-2[1]

PERCENTAGE OF A CRITICALLY HARMFUL IMPAIRMENT THAT IS REMOVED BY A SPECIFIED TREATMENT (ROW), LISTED BY AFFECTED USER CLASSES (COLUMNS)

USERS / TREATMENTS	1 Aesthetics and Recreation	2 Public Water Supplies	3 Aquatic and Wildlife	4 Agriculture: Home	4 Agriculture: Livestk.	4 Agriculture: Irrig.	5 Industry[2] (See Each Industry)	6 Navigation	7 Hydro Power	8 Waste Disposal[3]
Preliminary										
Screening--Coarse and Medium								Removes floating solids and some oil	See 6	
Fine Screen										5-10 BOD; 2-20 SS; 10-20 Bac.
Primary										
Plain Sedimentation			35-40 BOD; 50 SS; 50 Tur.; 25-75 Bac.		See 3					
Secondary										
Sedimentation + Activated Sludge		80-95 BOD, 95-98 Bac.; 85-95 SS		See 2		See 2				
Slow Sand Filter on low turbidity (not often used)	99 Bac.; 95-100 Tur.; 30 color; 100 odor/taste; 60 iron		See 1		See 1		See 1 (Bac. and Tur. features are less critical)			
Rapid Sand Filter + Coag. + MPN	90-99 Bac.; 100 Tur.; color under 5 mg/l; alkali and CO_2 up; Iron, odor and taste down some; Coag. affects pH									
Trickling Filter and Sedimentation		80-95 BOD; 70-92 SS; 90-95 Bac.		See 2						
Rapid Sand Filter + Coag. + Chlorine + Activated Carbon + MPN	(For Primary Contact Areas) 100 Bac.; 100 color, odor, taste, 100 Tur.; Iron and Manganese down; Coag. affects pH	See 1		See 1						
Chlorination (alone)		100 Bac.; color and odor partly down		See 2		See 2				
Tertiary										
Ion Exchanges or Reverse Osmosis or Electrodialysis		Demineralization, Hardness and TDS down; pH effect		See 2		See 2	See 2--Use here depends on raw water quality and use of own internal treatment facilities	See 2--Use here is to protect against corrosion	See 6	

TABLE III-2 (Cont.)

Notes:

1/ Sources are NTAC (26), Wolf and McKee (22), and those listed for Table III-1. Symbols and abbreviations used are:

SS = Suspended Solids
Bac = Bacteria
Tur = Turbidity
MPN = Most Probable Number of coliforms is less than 20,000/100 ml.
Coag = Coagulation
TDS = Total Dissolved Solids

2/ NTAC shows that it is not possible to deduce uniform standards for the boiler makeup, cooling and process water uses by all different industries. The entries here are thus meant as a rough indication of some generally common characteristics, namely that BOD, bacteria and turbidity do not seem to be critical contaminants except for leather and food/kindred products industries (which have virtually the same requirements as consumption), but there is concern for trace element removal.

3/ Waste Disposal has no specific quality requirement; any process that will reduce BOD and SS permits better waste assimilation.

"Sedimentation and Activated Sludge" shows that this treatment process affects three of the contaminants considered harmful to drinking water, namely BOD, bacteria, and suspended solids by reducing them in the respective amounts of 80-95 percent, 95-98 percent, and 85-95 percent. That such treatment alone is not sufficient to render "typical" raw water potable, however, is indicated by the fact that there are other entries in Column 2. These entries reflect the previously mentioned quality criteria and thus imply that the sedimentation/activated sludge process (1) does not remove the impairments it does affect completely enough, and/or (2) does not affect all factors critical to drinking water purity. For example, chlorination and some demineralization should follow sedimentation/sludge, as indicated by the Column 2 entries by those processes.

As a final explanatory note, for the use (column) in which an entry "See i" appears, this means that the description of what the designated treatment accomplishes is found in Column i, horizontally to the left. Thus, "See 3" in Column 4 (Agriculture-Livestock use) indicates that plain sedimentation treatment relieves the same impairments here that it does for Aquatic and Wildlife use (Column 3).

Table III-2 accomplishes one of the primary tasks of this project. That is, by knowing what processes are most generally needed to produce water quality acceptable for particular uses, one has at least an initial idea of what are the essential ingredients of a supply function that incorporates quality considerations. More precisely stated, it can be seen by reference to the classical economics model of Chapter II that the results so far provide some information about two of the elements which are critical to the empirical implementation of that analysis, namely the QIF h(q) and the production function $g(x_1, x_2, F)$. While specific forms for each of these functions must be derived (especially as regards $g(\cdot)$), this is, to some extent, a technological issue resolvable by reference to engineering sources. Unknown, however, are which items must be included in the technological formulations. For example, in considering water suitable for aquatic and wildlife preservation, sedimentation is probably adequate treatment. Still needed, however, would be information about sedimentation as a "production process" and reference to NTAC's recommendations about BOD, suspended solids, turbidity, and bacteria tolerances for this user class so that quality constraints representative of Equation (3) could be constructed. (Chapter IV will present an explicit form of quality constraint, while Volume Two will illustrate it by means of a numerical example which includes a stipulated production function.)

Another result desired from this project was an indication of how feasible it is to combine users into "economic classes" because of common treatments required (and hence, presumably similar treatment costs). Table III-2 shows that such grouping can probably be done only to a limited degree; quality criteria and critical impairments vary too much among the use classes to permit extensive "collapsing." The only two clear instances are the similarities between navigation and water power and the obvious case of agriculture-home use being virtually identical with public water supply use. In short, the 8- (now 7-) user classification constitutes a sufficiently high level of aggregation so that further grouping likely would not be expected.

A concluding word of caution is in order. Even though it is seen that water criteria can be segmented by user classes, this does not necessarily imply that the only way to handle water supply is by a distinct function for each user. Such an approach might, for pragmatic reasons (data availability, e.g.), prove to be the most efficient means of application, but here, at least conceptually, one can envision a single supply function with embedded quality parameters which permit measuring quality on a continuously increasing scale. Only when all the parameters a user deems critical "reach" his tolerance levels will the supply affect him.

Quantification of Quality by Means of Indices

Although all discussion so far is generally applicable to the formulation of Chapter II's QIF, either Form (1) or Form (2), the relevant comments of the preceding subsection are probably more directly concerned with Form (1), which can be viewed as a quality constraint for each relevant impairment. Recall that Form (2) visualized the QIF as an index number summarizing criteria on several water quality factors. The rationale for empirically implementing such a construct is very straightforward. All that needs to be done, in fact, is to interpret the I^* parameter in Chapter II's cost function derivations as an index number (regardless of how such is derived, so long as it is conceptually a measurement on final output of treated water). In other words, the purpose of Chapter II was to demonstrate that it is theoretically consistent to have a marginal cost function (Equation (10)'s MC), one of whose arguments is an explicit quality-related parameter I^*, even if the form of this parameter is only very broadly specified. Any index number based on output observations easily qualifies and (assuming data availability) would make Equation (10) empirically operational immediately for, e.g., regression analysis. That is, knowing that Equation (10) does exist conceptually, one merely has to decide on a form for the cost functions, without considering (as is often the case in economic analyses of "reduced form" situations) the form of the underlying production function. Econometric estimation of Equation (9) or (10), therefore, could simply involve different regressions of treatment costs on input prices, output, and quality index.

At present, the literature contains three hypothesized (with sample calculations) water quality indexes amenable to the procedures outlined above. Listed with abbreviated descriptions of how they are constructed (sources with fuller details are cited), these are:

(1) Mitre Corporation's PDI [developed for the Office of Water Programs, Environmental Protection Agency (EPA); see Mitre (25) for details about the index]--"PDI" stands for "Prevalence-Duration-Intensity," these terms representing, respectively, the number (P) of miles of uniformly polluted water in a reach; the portion (in quarters, as an index D) of a year that the pollution exists; and a composite ecological/utilization/aesthetic index I of the extent of the pollution.[12/] The D and I components are numbers between 0 and 1, so that when the index is formed as the product P•D•I, it is in actuality a "weighted miles of length" measure, higher values of which indicate a more severe pollution problem. Mitre recognizes this feature as one of PDI's drawbacks because high values of D and I applied to a low value of P could result in a PDI value smaller than that for a situation with low D and I/ but high P. In this case some persons might judge the former instance to be the more severe problem. The index must

12/ Mitre defines these components: I_1 (ecological) = index of how damaging to life the pollution is; I_2 (utilization) = index of how disruptive to normal uses the pollution is; I_3 (aesthetics) = index of visual and odor unpleasantness. Permissible values which translate verbal into numerical responses are specified for each component so that opinions (see text) can readily be rendered in numeric form for each I_i. Summing then gives $I = I_1 + I_2 + I_3$.

therefore be used carefully, probably having least ambiguous meaning when applied to reaches of similar length.

As indicated, P is simply measured in miles, while D is a time measure; accordingly, both are data observations. On the other hand, I is subjectively determined by experts' opinions. In the Mitre study, regional offices of EPA provided estimates of PDI for specified water bodies.

Although PDI can be considered an operational first-hand water planner's tool, the Council on Environmental Quality (CEQ (6, p. 12)) has noted some weaknesses aside from the "mileage dominance" trait cited above. Perhaps most serious is the fact alluded to above that even though data on water quality conditions exist, there is no assurance that they have been used by EPA personnel in their judgmental estimates of I. Even if such information has been used, there is no indication of a uniform and systematic manner which could be replicated. This trait points out a second weakness, namely that PDI does not explicitly account for pollutant types. Thus, for example, when a person whose opinion has been solicited assigns I_1 = 0.2 ("conditions that produce stress on indigenous life forms"), he has internally synthesized his knowledge about pH, turbidity, BOD, etc. factors and decided that of the five conditions Mitre allows to be assigned to I_2, a value corresponding to I_2 = 0.2 is most representative of the watercourse being evaluated. The quality factors themselves do not appear in PDI. Finally, CEQ does not feel that PDI is "...sensitive enough to detect trends except after several years."[13/]

(2) <u>Syracuse University Civil Engineering Department's PI</u> [see Syracuse ....(29) for details] --In a sense at the other end of the spectrum from PDI, Syracuse's PI ("Pollution Index") is based on explicit consideration

13/ CEQ (6), p. 13.

of 14 monitored water quality factors.[14] For a specified j^{th} water use, one needs observations on (1) the current measured level C_{ij} of each i^{th} impairment, and (2) the recommended tolerance L_{ij} by Use j of each Impairment i. The dimensionless ratio $R_{ij} = C_{ij}/L_{ij}$ thus indicates how critical the level of Impairment i is in Use j; a value exceeding one says treatment is needed because the monitored level exceeds the tolerable level.

The PI_j for Use j is formed by first computing R_{ij} for the 14 listed quality parameters. As in indication of the "average" impairment concentration, the mean $\bar{R}_j$ of these values is calculated. But "...the average value may not satisfactorily measure pollution, because the necessity of water treatment for a use is often determined by the maximum of the (R_{ij}) values rather than the average value."[15] This means that since any single R_{ij} value greater than one is a signal that treatment is needed, a mean value $\bar{R}_j < 1$ is misleading if interpreted to say that the water body is satisfactory. For this reason, Syracuse proposes that the maximum of the R_{ij} values, denoted max R_{ij}, be used along with $\bar{R}_j$ in computing PI_j.

The specific form ultimately used for PI_j is based on the geometric property of a two-dimensional space with co-ordinates $\bar{R}_j$ and max R_{ij}. Since the standard formula for the length of a radius vector in this space recognizes the influence of both values (and is of the same order of magnitude as them), this measure is adopted. Imposing certain boundary (normalizing) conditions, one arrives at a square-root-mean-of-squares formula:

14/ Temperature, color, turbidity, bacteria, total solids, suspended solids, total nitrate, alkalinity, hardness, chloride, iron and manganese, sulfate, and DO.

15/ Syracuse University, Department of Civil Engineering (29), p. 15.

$$PI_j = \sqrt{\left[\bar{R}_j^2 + (\max R_{ij})^2\right]/2} \quad . \tag{11}$$

Extended use of PI_j is hypothesized by proposing region-wide computations (for example, a weighted average of the Use j's in a region) as well as indices for groups of users (the Syracuse authors suggest three groups: human contact use, indirect contact use, and remote contact use[16/]). Finally, there are suggestions about how to compute R_{ij} for any parameters (e.g., pH), the observations on which are not recorded in usual quantity dimensions.

It is apparent that the PI formulation affords a greater degree of technical precision than does the PDI because it is "totally objective," thus correcting that major criticism of the PDI.[17/] On the other hand, this objectivity may well be a drawback, depending on the availability of the requisite quality factor observations, even assuming accurate means of monitoring have been employed.

In addition, a pragmatic water planner might bemoan the fact that PI does not take into consideration the size of, or time element relevant to, a water body being examined (in short, the "P" and "D" of PDI). That is, PI is essentially a "spot" analysis. An obvious suggestion to correct this deficiency, however, would be to use PI to compute the "I" of PDI, leaving "PD" as already empirically calculated. Such a "P·D·PI" index would be comprehensive in scope and virtually free of subjective aspects.

16/ Human contact would include uses like drinking, swimming, beverage manufacture; indirect contact would have fishing and agricultural uses, e.g.,; remote contact uses could include industrial cooling, aesthetics and navigation. Observe that this three-way classification is indeed a feasible higher level of aggregation than that discussed previously with respect to Table III-2, but it is achieved with compromise. Syracuse, for example, computes the average of the i^{th} impairment tolerances (L_{ij}) of the uses in a group and sets that as "the" tolerance on Factor i for the entire group.

17/ For this reason, it is interesting to note that CEQ (6) makes no reference even to indicate knowledge of the existence of PI.

(3) National Sanitation Foundation's WQI [see Brown, et al. (2) and (3) for details]--In a very real sense the Water Quality Index (WQI) is a combination of the subjective nature of PDI's "I" and the objectivity of PI just discussed. Briefly stated, it is constructed by collecting water experts' opinions about the severity of specified individual impairments in a water body and then synthesizing these responses into a single representation. To be more precise, in the examples shown, each expert was first asked (via questionnaire) to state which items from a list of quality factors he felt were most important to review in water analyses. From 94 (of 142 mailings) responses, a list of nine "most important" parameters was chosen for inclusion in the ultimate WQI: DO, fecal coliform count, pH, BOD, nitrates, phosphates, temperature, turbidity, and total solids. In a subsequent questionnaire, "...respondents were asked to assign values for the variation in level of water quality produced by different strengths of (the) nine...selected parameters"[18] by sketching a representative curve on a graph whose coordinates were "water quality" (as dimensionless numbers between 0 and 100) and "parameter strength" (measured in typical units for the parameter). The "average" of all the respondents' sketched curves for each factor was then deduced, to be used as a transformation function which translates any given parameter concentration into a quality level number (symbolically: quality number $q_i = f(m_i)$, where m_i is the measured parameter concentration). Finally, respondents' comments about the importance of the parameters relative to each other were

18/ Brown, et al. (3), p. 341.

used to construct parameter weightings w_i so that the nine individual quality level values q_i could be combined into one index, as[19/]

$$WQI = \sum_{i=1}^{9} w_i q_i \quad . \tag{12}$$

The procedure for applying (12) would thus be to obtain measurements m_i on each of the nine parameters ($i = 1, \ldots, 9$) for a watercourse, translate these observations into quality values by means of the $q_i = f(m_i)$ functions, and then sum these values, weighted by the w_i. The result is clearly a quasi-objective index.[20/]

WQI shares with PI the weakness that it does not incorporate "P" and "D" kinds of elements. Following the suggestion made earlier with respect to PI, however, a way to answer this criticism would be simply to replace "I" by "WQI" in PDI. Similarly, WQI, like PDI, has subjective content that some might criticize. However, it is of a more dispersed nature being interspersed with more empirical data. It may well be that, as a planner's tool, WQI's subjective content is an asset if it turns out to be the case that the "expert opinion determination" of w_i and q_i permits WQI to indicate more emphatically the severity of an acknowledged pollution problem than does the purely objective determination of R_{ij} in PI.

19/ It should be observed that WQI is proposed as a general "overall" index. That is, unlike the PI, there is no attempt to propose use-specific WQI's. This issue is not ignored, however, and Brown, et al. (2, pp. 8-14), cite evidence which compares WQI with two use-related indices and conclude that WQI rates water quality levels about the same as the two specific indices. Thus, a vote in favor of a general index is cast.

20/ It should be noted that a critical feature of the WQI derivation was the use of the DELPHI ("feedback") process in obtaining the experts' responses. This ensured a more systematic means of evaluating the received replies and thus gave greater assurance that the functions derived are "correct" depictions of the world.

By way of conclusion and reiteration, three extant formulations are presented which can play the role of I^* in Equations (9) and (10). All three are "Form (2)" QIF's, although the Syracuse PI can readily be taken as a Form (1) example since its formula preserves direct observations on the endogoneous variables of Chapter II's classical economics model. Regardless of which QIF form each represents, it is clear that each can be quantified and hence observations collected so that the "direct" regression analysis between costs and input prices/quantity/(quality consideration) outlined previously is feasible.

B. IMPAIRMENT TREATMENT COSTS

The second item for which observations are needed in order to quantify the theoretical analysis of Chapter II is treatment costs, especially unit impairment treatment prices. Cost information proved to be in the literature, although the precise form desired was not found in the sources surveyed. More specifically, cost particulars seem to fall into two categories, namely, simple historical cost tabulations and statistically-estimated cost functions. These will be addressed in turn, by citing source examples of each.

Cost Tabulations

Linstedt, et al. (20) have analyzed water treatment in Denver, Colorado, with a purpose very similar to the thrust of this study, namely associating treatments with stipulated levels of impairments. Their study is of a primarily descriptive nature, but it is unique in that it compares levels of quality factors in Denver's secondary treatment wastewater effluent with recommended tolerance levels for potable, irrigation and certain general industrial (boiler and cooling) water uses to ascertain how much upgrading would be required to make the secondary effluent suitable for these three purposes. The authors then proceed, as done here, to identify what tertiary treatment processes are generally associated with alleviating the undesired contaminants. Finally, they list costs (not necessarily specific to Denver) of the treatments. This cost information is summarized in Table III-3.

Because Table III-3 refers to tertiary treatments, the data presented there necessarily refer only to rather high quality raw water input. Consequently, these costs are probably lower limits for total treatment of ordinary raw wastewater unless there are very substantial economies in unit variable costs associated with the variable factors involved in treatment, such as chemicals. (That is, lower quality raw input will require more total chemicals, but a "quantity discount" lower unit price on chemicals, coupled with a larger volume of impairments, may likely cause lower cost per unit impairment treated.) Combined with the information on pre-treatment contaminant levels, it is possible to obtain an indication of cost per unit of impairment treated. For example, the authors record a "reuse-removal increment" of 155,000 coliforms/100 ml as the amount of bacteria that must be removed in order to raise Denver's secondary effluent to a quality level acceptable for irrigation purposes. Dividing this figure into the irrigation-bacteria treatment cost in Table III-3, one obtains $\$0.644 \times 10^{-7}$ to $\$1.288 \times 10^{-7}$ as the range of treatment cost (per 1,000 gallons treated) per "coliform concentration unit." Note that any such computation based on data where only two levels ("before" and "after" treatment) of impairment concentration are given necessarily implies a constant average cost interpretation. One cannot take issue with this result until detailed analyses to the contrary appear in the literature; the fact simply is mentioned for completeness.

TABLE III-3

TERTIARY TREATMENT COSTS OF UPGRADING DENVER SECONDARY WASTE EFFUENT *

Use	Main Problem	Treatment	Cost in Excess of Secondary Trmt. ($/1000 gal.)
Irrigation	Bacteria	Chlorination, Ponding	$ 0.01-0.02
Low psi boiler water	Mineral Impurities	Chemical Precipitation; Carbon Adsorption	0.10-0.11
Industrial Cooling Water	Minerals	Coagulation, Softening, Sedimentation	0.04-0.05
Potable Water (Total Tertiary Cost: $0.315-$0.425)	Nutrients	Algae Harvesting; Alum Coagulation, Other Biochemical Processes	0.05-0.15
	Suspended Solids	Filtration	0.035
	Organics	Carbon Adsorption	0.08
	Inorganics	Electrodialysis or Ion Exchange	0.14-0.15
	Pathogenics	Chlorination	0.01

* Summarized from Linstedt, et al. (20). There is no indication that capital costs have been excluded, so these figures are presumed to be total costs, that is, these are treatment costs which cover debt service in addition to operation and maintenance.

Probably the most detailed compilation of costs on a plant-by-plant basis is the result of a 1965 survey of 25 treatment plants by Louis Koenig (19). The study is both a data listing and a cataloging of some descriptive observations based on the survey data. It is the former that is of particular interest here because of its relatively detailed breakdown. Data are available on design factors (e.g., plant daily capacity), unit prices (investment, labor, energy, and chemicals), average raw water quality,[21/] unit input consumptions (manpower, energy, water, chemicals), and "total average" costs.

Because raw water quality factors are included along with treatment costs, some of which can be associated directly with certain impairments, Koenig's survey permits reference to tolerance criteria to make computations of some unit "impairment-level-alleviated" costs, similar to the example given above with respect to the Linstedt, et al. study. In fact, Koenig's data serve as the basis for quantifying the quality constraint of the classical economics model in Volume Two. The advantage of Koenig's data over Linstedt's is that they are more extensive and do not pertain only to tertiary treatment. Its major disadvantage is that, even as comprehensive as it is, it does not give information about all the steps involved in treating listed impairments. For example, no capital costs are associated with specific treatment steps, although such investment allocation may simply not exist since it was found nowhere in the literature searched. In addition, there is no correspondence between observations on contaminants and treatments for them. For example, raw water dissolved solids content is listed, but

21/ Although observations on turbidity, temperature, total alkalinity, pH, hardness, total dissolved solids, and color are recorded, Koenig (19, p. 300) notes a "...real deficiency of the data is in the small number of raw water quality parameters which are regularly measured at ... plants."

not a corresponding treatment; chlorine consumption is given, but not raw water bacteria content. Koenig does provide, however a good format to follow in collecting the kind of data which seem to be most needed for helping quantify quality in water supply analysis.

Two other cost tabulations surveyed bear mention, although they appear of less importance to the current study. Appendix D of Wollman and Bonem (31) gives data on (1) contaminant-removal efficiencies a percent of various processes for six kinds of impairments (dissolved (TDS) and suspended (SS) solids, BOD, COD, Nitrates and Phosphates), and (2) typical post-treatment impairment concentrations. Although not as straightforward as calculations based on Linstedt, et. al., or Koenig, one could use (1) and (2) to determine pre-treatment impairment levels and thus deduce impairment units removed by a process. Capital and operation/maintenance costs per million gallons treated per day are given in 1965 dollars for various treatment processes, and for five different capacity plants. The processes are listed at a higher level of aggregation than Koenig has (e.g., "high rate trickling filter" is listed, not energy and/or chemicals associated therewith), but this may well be a distinct advantage, assuming the figures given have already accounted for all relevant components. One feature of the data is that they show clearly that a single process usually affects multiple contaminants; hence, there is an inherent "allocation problem" in associating treatment costs with specific impairments removed. The data also indicate that there are definite scale economies in operation/maintenance functions.

Finally, Mitre Corporation (24) presents some data on operation and installation costs for a variety of treatments. No attendant information on impairment levels is given, however.

Derived Cost Functions

The other form in which the surveyed literature gave cost information relevant to this study is by means of estimated cost functions. One characteristic was uniformly true of all these sources: the functions derived are all of single variable form, to wit (symbolically) Cost = F (one characteristic). We believe that, while such results are good first-line guides, it would be more useful to have multivariate forms which consider several characteristics simultaneously. A forthcoming study purporting to do this will be referred to subsequently.

Robert Smith, now of the Robert Taft Water Research Center and one of the most oft-cited names on the subject of treatment costs, has estimated functions in Smith (27), each of which relates capital, operation/maintenance, or debt service cost for various processes to the millions of gallons per day (mgd) design capacity of a process. No equations are given, so one must read results from double-logarithmic graphs. Johnson (17, p. 45) has compiled some of Smith's tertiary ("advanced wastewater treatment"--AWT) results at the 10 mgd capacity level, and they are virtually the same as the figures in Table III-3 given previously.

Michel (23) has used data from 1,600 audits by the Federal Water Pollution Control Administration (FWPCA) of federally-assisted municipal wastewater treatment plants for extensive regression analysis. He has imposed the form $Y = aX^b$, which has the familiar log - linear transformation $\log y = (\log a) + b(\log X)$, as a basis for his regressions, where:

Y = annual cost (dollars or manhours)

X = { (i) average plant flow F (not capacity) as mgd; or
(ii) population equivalent PE, which is the sum of population and "industrial equivalent population" (based on a BOD conversion formula) served by a plant.

Each time regressions are run, they are done two ways, with X = F and X = PE. (Note: "mgd" = millions of gallons per day.)

Briefly stated, Michel's estimates are for nine types of treatment process and include (playing the role of Y) total, operation/maintenance (O/M), labor, electricity, and chemical costs, as well as man-hours needed for O/M. In each case, unlike Smith, Michel gives his estimating equations and records the simple correlation coefficients between logarithms of Y and X so that regression coefficients of determination ("R^2" in more familiar terms) can be calculated. In no case does he obtain $R^2 > 0.81$, and for many equations R^2 is considerably lower. Furthermore, he nowhere records levels of significance for the estimated coefficients ($\widehat{\log a}$) and $\hat{b}$. Thus, his results are perhaps not as strong as would be desirable from an econometric viewpoint, but his efforts constitute a systematic approach to cataloging costs at a level of aggregation which, again, may prove to be the most useful for pragmatic purposes.

Hinomoto (14) has run regressions on Koenig's (19) data, based on the form (again, easily linearized into logarithms) $(C/K) = aK^{(b-1)}$, where (C/K) is a cost per unit of design capacity K. He estimates equations for capital investment, total chemicals, pumping and heating engines, manpower, maintenance and repair, and miscellaneous items. All but one of his R^2 values are below 0.6, and he also gives no coefficient significance. Regardless of statistical strength, however, Hinomoto's results are of limited use for this study because there is no identification of treatment process.

Using some linear (or log-linear) univariate regressions and some non-linear regressions of the form $y = 1/(ax + b)$, the Dorr-Oliver Corporation (8) has estimated capital costs as functions of different variables for conventional and tertiary processes. In addition, they have estimates of operating costs for some specialized activities (e.g., sludge combustion). Their data base is unique in that it has been compiled from firm sales and engineering records, but the overall usefulness of such a capital cost component breakdown for this study is probably of limited value, since what is most needed (recall "F" in the classical economics model) is cost information on entire plants as "packages."

We conclude this description of cost studies surveyed by mentioning an in-progress study that will likely prove to be of most use to the purpose of this project. Mr. Richard Kaczmarek, a Senior Associate Engineer in the Environmental Programs Department of the IBM Federal Systems Center, is currently a doctoral candidate in Environmental Engineering at the West Virginia University. His dissertation is entitled "Construction Costs of Municipal Sewage Treatment" and purports to be a rather comprehensive study. Two of its major contributions will be the use of single-equation multiple regressions ("...up to 80 independent variables") and the use of new detailed and updated data. In a telephone conversation, Mr. Kaczmarek noted that he intends to examine not only construction costs but also operation/maintenance costs, by treatment steps. To date he has collected most of his data (secured by survey) but has not run any regressions.

C. PRODUCTION FUNCTION FORMS

The third and final datum needed to implement Chapter II's model is a characterization of a treatment plant production function. The technical listerature surveyed yielded nothing usable, the relevant information generally taking the form of engineering equation representations of the physical principles underlying individual processes. Attempts at deriving an engineering-economic function as suggested by Vernon Smith (28) were also fruitless. For this reason, the use of regression analysis applied to hypothesized forms and available data is advocated. At this point, such an approach is believed to be the most operationally efficient means of deducing the "black box" representation needed. Some sample results will be presented in Volume Two.

IV. METHODOLOGY FOR INCORPORATING QUALITY ASPECTS IN WATER SUPPLY FUNCTIONS

The function of this section is to describe how the data sources of Chapter III can be used to quantify quality in water supply. Chapter II's classical economics model (CEM) will be examined first, and then a mathematical programming model to demonstrate resource allocation applications will be presented.

A. CLASSICAL ECONOMICS MODEL

Explicit, Measurable Quality Constraint

Of the three items requiring quantification in CEM, Equation (3)'s quality constraint is probably the most critical, because it can affect the form in which cost data are required. That is, the form in which impairment levels (and possibly treatment quantities as well) appear may dictate precisely what unit prices one must use.

There are two ways to quantify Equation (3), corresponding to Chapter II's QIF Forms (1) and (2):22/

(1) Similar to the construction of the Syracuse PI, one can render Equation (3) in terms of the components that comprise a quality index. That is, one writes a quality constraint itself rather than deducing a quality index per se. Toward such end, consider the following formulation, again cast in general terms of the one-impairment CEM.

22/ Recall Equation (3): $I^* = h(g) = h(g[x_1, x_2, F])$.

Let

$m =$ (known) amount of composite treatment input needed to treat one unit of "composite" impairment factor. In the practical application developed here, m will be assumed constant, to facilitate empirical operationality. (This imposes a "fixed proportions" relation between an impairment and its treatment, but if available data required relaxing this stipulation, it could be done easily without affecting the conceptual results presented.) Thus, m has the dimensions (treatment input amount)/(impairment amount removed), and the ratio (x_2/m) is the total amount of impairment factor removed by x_2 quantity of treatment input.23/

$n =$ (known) amount of impairment typically present per gallon of raw input water (i.e., the usual pretreatment impairment concentration). Thus, (nx_1) is the total impairment amount present before treatment. A "fixed proportions" relation such as that described above for m is assumed assumed here, therefore, between impairment amount and input water.

Now, a very reasonable constraint representation is to depict post-treatment impairment concentration as a specified level; that is,

$s =$ Post-Treatment Impairment/Output (e.g., pounds of turbidity per million gallons of treated water)

$$= \frac{\text{(Pre-Treatment Impairment)} - \text{(Removed Impairment)}}{q};$$

Thus

$$s = \frac{nx_1 - (x_2/m)}{q}, \quad \text{where } m \text{ and } n \text{ are exogenous} \qquad (3')$$

parameters, observations on which would come from plant data; and the parameter s is a contaminant tolerance recorded in, most likely, NTAC (26). Thus Equation (3)'s quality constraint is here rendered as a specified (maximum) amount of residual impairment that will be tolerated in water that has been treated. Once the output production function is substituted for q, (3') becomes a function of

23/ $(x_2/m) = x_2 \cdot (m)^{-1}$

$$= \text{(treatment input amount)} \cdot \frac{\text{(impairment amount removed)}}{\text{(treatment input amount)}}.$$

only the decision variables x_1, x_2, and F so that optimization proceeds as described before. Volume Two's illustrative examples show how this form of quality constraint is implementable and leads to cost functions in which s appears explicitly.

(2) One can attempt to define an actual h(q) QIF function by determining a relation between values of a quality index and q. One of the most practical means of doing this would be to collect observations from many plants on one of the three indices described in III-A, along with observations on the corresponding output q values. As a cross-section sample, these observations would be directly amenable to regression estimation. With a QIF in hand, it then becomes a straightforward matter to substitute, as done in (1) above, whatever production function form has been adopted and proceed with the optimization steps presented earlier.

Quantification of either the (1) or (2) form of Equation (3) would not be difficult, provided data have been secured. For (2), plant level observations on a quality index are required, while (1) can be implemented with data in the detail given by Koenig. However, it would not be surprising if (2)'s data requirements (e.g., collect raw data, construct quality index observations, determine QIF) proved more time-consuming and difficult to satisfy than (1)'s (merely collect raw data and compute (3') as shown).

On the other hand, (2) is appealing because it represents a means of accounting for several quality factors simultaneously, while (1) suggests a constraint for each factor. The mathematical complexity of the latter multi-constraint problem is forbidding but not impossible to handle, as will be demonstrated in Volume Two.

There will be no attempt in Volume Two to implement (2) because of an absolute lack of plant level observations. From Koenig's data, however, we have been able to construct an illustrative example of (1) which, when combined with an estimated production function, permits deriving a complete cost function.[24/]

Use of Treatment Cost Information

Turning now to the question of implementing cost observations, there is little descriptive information that can be added to the data citations in Chapter III, but some interpretative amplification is in order. In particular, the information presented there indicates the CEM's x_2 (treatment input) can have either a "macro" or a "micro" interpretation. The former would give x_2 dimensions of, for example, "secondary treatment amount per year" (very macro) or "amount of filtration per year" (moderately macro), while the micro would view x_2 as, for example, "pounds of chlorine per year." Correspondingly, then, the unit price w_2 would have to be measured as, respectively, "\$/unit of secondary treatment," "\$/unit of filtration," or "\$/pound of chlorine." The majority of the cost sources surveyed, it was seen, have macro rather than micro level data, and it is probably true that, short of collecting one's own more detailed data, this is what is most likely to be found in the published literature. With such data more readily available, there would be substantial incentive to favor the more aggregate level interpretation/implementation of CEM. The case for macro interpretation grows stronger when you include the fact that individual treatments often affect several impairments (which would mean also that fewer Equation (3) quality constraints would have to be specified).

24/ Note that the analog of quality index I^* is the parameter s in the case of (1) here, where low values of s reflect higher quality requirements. The cost curves of Figure II-1 would thus be indexed on s, higher curves now corresponding to lower s values.

These macro advantages come at a cost, however. To begin with, the macro measures of x_2 may not be easy to define, much less obtain observations for. For example, how does one measure the "amount of filtration (services) used per year?" Problems of measuring capacity utilization and of combined stock-flow measures quickly arise.[25/] Second, it is almost certain that a macro interpretation would require using a Form (2) index number QIF in order to express a collective quality constraint consistent with the several impairments being alleviated by the aggregate treatment process. The previously-described problems attendant to implementing any of the quality indices would consequently be encountered. Finally, the macro approach precludes using the analyses here to estimate costs of alleviating specific contaminants (this may not be a disadvantage at all if one is merely concerned with deducing incremental costs of quality, where "quality" measured as a summary index is perfectly acceptable to the water planner).

Thus, it is clear that there are advantages and disadvantages to choosing either the micro or macro interpretation of CEM. Our illustrative example in Volume Two is a micro view, but this has been dictated by data availability rather than by a decision that such is "the correct" position.

Guides for Implementation and Generalization

In concluding this section, we reiterate that CEM is the basis for cost functions, at least conceptually. It will be seen in Volume Two, however, that, even with the straightforward QIF of Equation (3'), the ability to derive explicit cost functions depends very critically on the

25/ Some additional effort is also required to put cost data into usable form. Let x_2 = filtration services per year. Now, let c_2 = ¢/1,000 gal. filtration cost as given in macro sources. In order to use c_2, however, one needs to define and measure u^{-1} = filtration services/1,000 gal., so that the unit price of filtration can be correctly given as $w_2 = c_2 u$, which has dimensions of ¢/unit of filtration services.

form of the $g(x_1, x_2, F)$ production function used. Even relatively simple forms for $g(\ )$ lead to optimization Equations (5) - (8) that are unsolvable by ordinary algebraic means. Also, it hardly needs mention that generalizing CEM to allow for more than one quality constraint adds greatly to the complexity.

Nonetheless, if an analytic determination of Equation (9)'s cost function[26/] is possible, then econometric estimation of its parameters is possible (these cost function parameters will be direct functions of the underlying production function parameters, which thus precludes the need to estimate $g(\ )$'s parameters separately). Should an analytic determination be impossible, then estimates of parameters of an hypothesized form for C_{LR}, as suggested previously, offer the only course of action. These notions will be made more concrete in Volume Two.

B. MATHEMATICAL PROGRAMMING MODELS

We conclude the analysis of Volume One by looking at water quality considerations in relation to resource allocation models. That is, attention is directed away from "exact" determination of supply functions in the traditional economic sense of CEM to an analysis of water quantities destined for various uses (hence the term "allocation") throughout an entire system (e.g., geographic region).

Conventional Models

In the literature on water resources management, "systems analysis" is one of the most prevalent topics discussed. And, within this topic, linear programming (LP) is perhaps the most frequently employed mathematical tool.

26/ Recall Equation (9): $C_{LR} = \varphi(\text{prices}; q; I^*)$, where "prices" refers to inputs.

Briefly stated, the linear programming model is a constrained optimization approach (just as CEM was), the term "linear" conveying the fact that objective functions and constraints are linear functions of the endogenous variables. With respect to water resources, a variety of LP applications and formulations have appeared in the literature, as surveyed by Drobny (9). In those cases where water quality aspects have been dealt with, Drobny points out that the usual approach is to analyze water users who discharge degraded effluent but subject to certain constraints. Generally only two parameters are considered, namely, BOD removed as an index of waste treatment used and DO as the index of water quality. "The problem in its simplest form reduces to finding the degree of BOD removal required for each waste discharger [along a water course] that will maintain specified minimum levels of dissolved oxygen in the receiving waters at a minimum total cost."[27/] Measuring how supply is affected by quality considerations is therefore a matter of conducting a sensitivity analysis to trace quantity changes that result when different DO levels are specified.

Assumptions and Formulation of New Model

In an attempt to broaden the applications of LP to water quality questions, we have formulated a programming model (whose objective function is profit-maximization rather than cost-minimization) which incorporates quality considerations in a different way. Thus, the perspective here is to center attention on producers who have a choice between procuring high quality water and procuring low quality water that they themselves will upgrade to acceptable (usable) quality. Furthermore, it might be possible to "downshift" (divert) high quality water to uses normally permiting lower

27/ Drobny (9), p. 1186.

quality.[28] In short, the question addressed here is that of ascertaining internal producer behavior with respect to using water of varying qualities rather than the more common issue of producer response to imposed effluent standards. The term "supply" thus refers here (as it did in the case of CEM) to "effective" supply (i.e., treated water) which is created by the user himself rather than by a separate facility whose sole purpose is water treatment (the latter was the case for CEM). The effect of quality considerations is therefore registered by measuring the impact of changes in certain quality-related parameters on producers' water use and output activities, as is described below.

Turning, then, to a formal (but reduced in dimension) presentation of the model, consider two output activities ("a" and "b") that use water of two qualities ("1" = high quality; "2" = low quality), and define these symbols:

p_a, p_b = parametric "net" output prices, for determining revenue net of non-water costs

q_a, q_b = output quantities per time period

x_{ij} = total gals. of water used at Quality i level per time period in producing Output j (i = 1, 2; j = a, b)

$$= \left(\begin{array}{l}\text{gals. direct use}\\ \text{from Quality i's}\\ \text{natural source}\end{array}\right) + \left(\begin{array}{l}\text{gals. of Quality 2}\\ \text{that are upgraded}\\ \text{to Quality 1; if}\\ \text{i = 1; or, gals.}\\ \text{of Quality 1 water}\\ \text{that are diverted}\\ \text{to Quality 2 use;}\\ \text{if i = 2}\end{array}\right) + \left(\begin{array}{l}\text{gals. of Qual. i}\\ \text{water previously}\\ \text{recovered for}\\ \text{reuse}\end{array}\right)$$

28/ This sort of usage would not be expected to occur unless the lower grade water were inadequately available, prohibitively expensive, and/or (if there were more than two quality levels) it were unfeasible to upgrade water on an even lower quality. The feature has been included here to help make the model comprehensive.

$$= \quad x'_{ij} + \begin{Bmatrix} \vec{x}^{j}_{12}; \text{ if } i = 1 \\ \text{or} \\ y^{j}_{21}; \text{ if } i = 2 \end{Bmatrix} \quad + \quad \tilde{x}_{ij}$$

c'_{ij} = \$/gal. procurement and use cost of Quality i water direct from natural source for use in Output j (unit cost for x'_{ij} and y^{j}_{21})

$\vec{c}^{j}_{12}$ = \$/gal. use and upgrade cost for changing Quality 2 to Quality 1 (unit cost for $\vec{x}^{j}_{12}$) in Output j production

$\tilde{c}_{ij}$ = \$/gal. use and recovery cost for wastewater to reuse as Quality i in producing Output j (unit cost of $\tilde{x}_{ij}$);

where it has been presumed possible only to upgrade Quality 2 water to be Quality 1, not vice versa ($\vec{x}^{j}_{21}$ is not defined); and similarly only diversion from Quality 1 to Quality 2 is possible (there is no y^{j}_{12} variable).

In terms of these variables, total joint profit per time period for both production activities is ("revenue" is net of non-water costs):

$$\begin{aligned} \Pi = (\text{revenue}) &- (\text{a's cost of water used as Quality 1}) \\ &- (\text{a's cost of water used as Quality 2}) \\ &- (\text{b's cost of water used as Quality 1}) \\ &- (\text{b's cost of water used as Quality 2}) \end{aligned}$$

$$\begin{aligned} \Pi = (p_a q_a + p_b q_b) &- (c'_{1a}\, x'_{1a} + \vec{c}^{a}_{12}\, \vec{x}^{a}_{12} + \tilde{c}_{1a}\, \tilde{x}_{1a}) \\ &- (c'_{2a}\, x'_{2a} + \tilde{c}_{2a}\, \tilde{x}_{2a} + c'_{1a}\, y^{a}_{21}) \\ &- (c'_{1b}\, x'_{1b} + \vec{c}^{b}_{12}\, \vec{x}^{b}_{12} + \tilde{c}_{1b}\, \tilde{x}_{1b}) \\ &- (c'_{2b}\, x'_{2b} + \tilde{c}_{2b}\, \tilde{x}_{2b} + c'_{1b}\, y^{b}_{21}) \end{aligned}$$

which can be rearranged (with some grouping of terms) as:

$$\Pi = \sum_{j=a,b} p_j q_j - \sum_{i=1,2} (c'_{ia}\, x'_{ia} + c'_{ib}\, x'_{ib}) - \sum_{i=1,2} (\tilde{c}_{ia}\, \tilde{x}_{ia} + \tilde{c}_{ib}\, \tilde{x}_{ib}) \quad (13)$$

$$- \sum_{j=a,b} \vec{c}^{j}_{12}\, \vec{x}^{j}_{12} - \sum_{j=a,b} c'_{1j}\, y^{j}_{21}$$

$$= (\text{revenue}) - \begin{pmatrix}\text{procurement}\\ \text{costs}\end{pmatrix} - \begin{pmatrix}\text{recovery}\\ \text{costs}\end{pmatrix} - \begin{pmatrix}\text{upgrade}\\ \text{costs}\end{pmatrix} - \begin{pmatrix}\text{divert}\\ \text{costs}\end{pmatrix}.$$

It is now desired to maximize Π with respect to q_j, x'_{ij}, $\bar{x}^j_{12}$, y^j_{21}, and $\tilde{x}_{ij}$ (i=1,2; j=a,b); subject to the following constraints:

Two Technological Constraints on Outputs: $q_j \leq T_j$ (14)

Natural Endowment Constraints on Available Water: (15)

$$x'_{1a} + x'_{1b} + y^a_{21} + y^b_{21} \leq E_1 \quad \text{, for Quality 1}$$

$$x'_{2a} + x'_{2b} + \bar{x}^a_{12} + \bar{x}^b_{12} \leq E_2 \quad \text{, for Quality 2}$$

Four Recovery Constraints for Reuse Water: (16)

$$\tilde{x}_{ij} \leq g^j_{i1} \;(x'_{1j} + \bar{x}^j_{12} + \tilde{x}_{1j}) + g^j_{i2} \;(x'_{2j} + \tilde{x}_{2j} + y^j_{21})$$

where g^j_{ik} is the portion of water that was used as Quality k in Production j and is now recovered as Quality i water for reuse.

Four Input/Output Relations: (17)

$h_{ij}\; q_j - (x'_{ij} + \tilde{x}_{ij} + u_{ij}) \leq 0$; where h_{ij} is the minimum number of gallons of Quality i water required per unit of Activity j production, and $u_{ij} = \left\{\begin{matrix} \bar{x}^j_{12} & \text{for } i = 1 \\ y^j_{21} & \text{for } i = 2 \end{matrix}\right\}$.

<u>Considerations Affecting Implementation</u>

Although numerical examples will be given in Volume Two, some amplifying comments are in order here:

(1) The endogenous variables x_{ij} measure water supply explicitly, while the individual components (which are also part of the LP solution) show the composition of each quality's supply.

(2) The primary quality-related parameters in the model are the various unit cost coefficients (particularly the $\bar{c}^j_{12}$ upgrade cost), the h_{ij} input/output coefficients, and the g^j_{ik} recovery parameters.29/

29/ It is clear that the endowment constraints of Inequalities (15) also incorporate quality aspects, but these characteristics are not likely to be changed by producers themselves. The importance of disturbing E_i in a sensitivity analysis of drouth, for example, is fully acknowledged, however.

Changes in values of these factors can be made to reflect different institutional or economic policies (through the unit costs) and/or technological alternatives (registered in h_{ij} and g^j_k—for example, increasing h_{1a} signals a more stringent quality requirement since this would indicate more gallons of Quality 1 water are needed per unit of output from Activity a). By proposing different parameter configurations, water use and output responses can be calculated, the former being most important to the water supply question.

(3) As regards empirical implementation, data observations on the indicated parameters (i.e., everything other than the endogenous decision variables being maximized with respect to) would be required. In general, a source similar in detail to Koenig (19) should be able to provide unit cost values, although there would need to be prior decisions about the definitions of "quality" so that the appropriate costs could be determined (for example, would quality be defined in terms of QIF values?). Parameter observations for Constraint (15) would require, for instance, reservoir level and/or flow information, while Constraints (14), (16), and (17) are technologically related to the production process. In this regard, it needs to be pointed out that (17) imposes explicitly the "fixed proportions" input/output production relation that is inherent to LP models. (In other words, estimating values for the h_{ij}'s corresponds to estimating CEM's production function given in Equation (2) of Chapter II). Similarly, Inequalities (16) assert a proportional relationship between recovered water and total water used. To be thorough and rigorous, the validity of these proportionality assumptions should be checked.

(4) Although the term "output activity" has been used here, it is capable of broad interpretation, depending on the scope of application intended. Thus, output q_a might represent all agricultural production for a region. In cases such as public water supply where a water use does not lead to some specifiable output, the model can be adapted by taking the water treatment process itself as the productive activity. Thus, h_{ij} would characterize the treatment "plant," p_j would be the unit price of treated water, and it might be that only upgrade variables like $\bar{x}^j_{12}$ are relevant.

For Volume Two's numerical illustrations, dummy data have been generated for the needed parameters in such a way that they reflect reasonable relative magnitudes. Unlike the single-impairment CEM which could be estimated, available data proved inadequate to characterize the linear programming model. The LP results presented in Volume Two demonstrate, however, that the model is mathematically consistent and lends itself to a variety of parameterization sensitivity analyses.

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B. SOURCES CONTACTED REGARDING WATER QUALITY AND TREATMENT COSTS

From the U.S. Environmental Protection Agency (EPA)

Mr. Frank Bell noted that treatment comes as a package, so it is difficult to separate aspects specific to certain impairments; also, there is little "hard" evidence about the effect of conventional water purification treatment on trace elements.

Dr. Dan Sokoloski (economist) cited general environmental works by EPA and the Council on Environmental Quality and referred to other EPA personnel.

Dr. Roger Schull (Research & Monitoring) cited the National Sanitation Foundation's WQI and Robert Smith's work on treatment costs.

Dr. Ralph Luken gave the same information that Dr. Schull did.

From the U.S. Department of Agriculture (USDA)

Joseph Biniek (per Dr. Grano per Dr. Cotner) indicated that most of their work is specific to soil characteristics. He sent a paper, "Planning Natural Resource Development," that has a cost/benefit analysis bibliography. He also referred to "North Atlantic Regional Water Resource Study" (we requested a copy from John Green in the Upper Darby office in Pennsylvania) as possibly having an LP model with visual constraints, but we have not yet received a copy.

From the IBM Federal Systems Center

Richard Kaczmarek has an in-progress dissertation that hopes to be linear regression analysis of wastewater treatment costs, by process steps. Data have been collected but no regressions have been run yet.

From the Mitre Corporation

Richard Greeley stated that the "I" of "PDI" was evaluated by experts' opinions. Details of regional calculations can be seen at Mitre. (He cited Al Erickson of EPA who may also have copies of computations and perhaps raw data as well.)

From the American Water Works Association (AWWA)

Ralph Uhlenburg (N.Y. Office) was solicited for any citations on treatment costs the AWWA might have. He sent a copy of its 1966 staff report, "The Water Utility Industry in the U.S.," but it proved to consist of aggregated data on future capital funds needed, financing means, etc., and had nothing on treatment costs.

From the National Bureau of Economic Research

Dr. W. J. Leininger called in October 1972 and learned there had been no progress on the Bureau's study of industrial water use. (A subsequent seminar in January 1973 sponsored by the Corps of Engineers produced a very preliminary report but indicated substantial work remained before results could be used by others.)

From National Sanitation Foundation

President Robert M. Brown sent a copy of "A Water Quality Index-Crashing the Psychological Barrier" which outlines the essence of the Foundation's WQI.

VOLUME TWO

SUPPLY FUNCTION IMPLEMENTATION: ILLUSTRATIONS AND APPLICATIONS

VOLUME TWO

TABLE OF CONTENTS

I. INTRODUCTION AND REVIEW OF DATA REQUIREMENTS FOR EMPIRICAL OPERATIONALITY

This volume constitutes the second volume of a three-volume study for the U.S. Army Corps of Engineers Institute for Water Resources (IWR) under Contract No. DACA 71-72-C-0053. It presents illustrative examples of, and implementation instructions for, the methodology developed in Volume One on incorporating quality factors into water supply analysis. In Volume One, two "supply side of the market" models were constructed:

(1) A classical economics model (CEM) demonstrated how the classical derivation of an economic cost function can be extended to show that a "quality indicator function" (QIF) can be embedded in a water treatment plant cost function. This enables computing the marginal cost of incremental amounts of water contamination (as measured by the QIF).

(2) A linear programming (LP) model was used to depict resource allocation aspects of water supply by analyzing multiple water users who are able to use water of different qualities when they themselves upgrade lower qualities to acceptable higher quality levels. Thus, the LP model measures effects of quality on water supply by gauging how a user changes the "effective" usable supply facing him in response to changes in different

quality-related parameters such as upgrade treatment costs.

Both of these models will be examined here, with special attention focused on what must be done to make each one empirically operational as a water planner's tool. In addition, techniques for generalizing each model are discussed in an attempt to render the analyses more broadly applicable.

A. CLASSICAL ECONOMICS MODEL (CEM)

Volume One discussed the economic theory basis for the importance of marginal costs within a treatment plant's optimizing framework. This led to the development of a constrained cost-minimization model, as given by Volume One's Equations (1)-(3) and re-presented below in this volume's Chapter II. That is, it was seen that the "classical" derivation of a process cost function is to minimize the cost of producing any specified output level and then trace the locus of such minima for all feasible outputs. Differentiation of this function with respect to output then gives marginal cost. In the present application, an additional constraint to incorporate water quality considerations has been devised as a means of enabling explicit measurement of costs associated with different levels of water impairment.

Schematically, the model minimizes total treatment costs (for which unit treatment input prices must be known parameters), subject to constraints reflecting (1) a desired output flow of treated water coming from a specified process (which is characterized by a known production function), and (2) a stipulated "quality" of the treated water output (as measured by values of the QIF). The results of the optimization process were seen to be total and marginal cost functions in which a QIF parameter appeared explicitly, as in Equations (9) and (10) of Volume One.

Production Function and Input Price Parameters

In order to make the CEM empirically operational, it is necessary to have observations on input prices, the production function, and QIF. Chapter IV of Volume One cataloged the kind of information needed. In brief, production function parameters would have to come from engineering literature or else be deduced from an estimating process such as regression analysis. Input prices, depending on the QIF, would need to come from a detailed source such as Koenig (6).[1/] For both of these items, collecting the required information should be relatively straightforward, assuming the data exist. Quantifying the QIF, on the other hand, is not quite as direct, because prior work must be done to formulate the QIF itself in such a way that it is manageable, both mathematically and numerically. In other words, the QIF must not only be couched in terms of output as indicated by the function h(q) described in Volume One, but it must also be done in a manner that enables measurement from available data.

QIF Development From the User vs. Treatment Classification

With respect to quantifying the QIF, it can be noted that a central purpose of Volume One's Table III-2 ("Users vs. Treatments") was to summarize critical information about types of treatment processes needed to alleviate impairments of special concern to specific users. Accordingly, a Table III-2 type of presentation would be consulted to determine what kinds of pollutants need to be covered by a QIF for a particular user class. In addition, by knowing the type of process intended to treat the pollutant, one also knows

1/ As in Volume One, parenthesized source citations refer to numbered bibliography listings appearing at the end of this volume.

more about the inputs on which observations must be obtained. For instance, in the single impairment illustrative example to be presented below in Section II-A, the QIF is stated as the amount of turbidity remaining (after treatment) per million gallons of treated water. By consulting Table III-2, Column 1, it can be seen that an effective treatment for turbidity is coagulation. Finally, since engineering sources and other information (e.g., Koenig's (6) data) indicate that there are certain bulk chemicals like alum that are used specifically as coagulants, one can thus anticipate obtaining "pounds of coagulant" as one of the input observations requisite to the implementation of this form of the CEM (details below will clarify this discussion).

Implementing an index number form of QIF is not as readily accomplished, mainly because the three index numbers surveyed in Volume One are not directly related to output flow. As described there in Chapters III and IV, however, there are two ways to deal with this problem: (1) deduce a functional relationship between the index number and output, and then use this as $h(q)$ in a complete CEM cost function derivation; or (2) elect not to derive a cost function analytically but rely instead on the knowledge (conceptually demonstrated) that a relationship among treatment costs, input prices, output and quality index does exist, and use this as a basis for estimating a cost function directly.

To illustrate these points, recall the Mitre Corporation (8) "PDI" index, which was seen to be made up of the components: P = prevalence (reach length), D = duration (proportion of a year that pollution is present), and I = intensity (the sum of three dimensionless fractions that are index number representations of observations on ecological (I_1), utilitarian (I_2)

and aesthetic (I_3) characteristics of the water body in question). Suppose that a water body 10 miles long is known to experience pollution for nine months of the year. These objective observations are translated into, respectively, the numbers P = 10 miles and D = 0.8 (dimensionless) in the Mitre formulation. Continuing, assume that subjective determination gives observations which are then transformed by Mitre's scales into the following values: $I_1 \doteq 0.5$ ("conditions that eliminate one or more indigenous life forms"), $I_2 = 0.2$ ("conditions that intermittently inhibit realization of some desired and practical use or necessitate use of an alternate source"), and $I_3 = 0.2$ ("visually unpleasant with unpleasant tastes or odors"). Adding, $\bar{I} = I_1 + I_2 + I_3 = 0.9$, so that the complete index for the hypothetical water reach in question is $PDI = 10 \cdot (0.8) \cdot (0.9) = 7.2$.

In order to apply this PDI notion to the CEM cost function concepts, observations would have to be recorded at various treatment sites, and the water reach measured in each case would have to be associated with a flow of treated water (q, in CEM terminology). When this is accomplished, either method outlined above for rendering an index number as a QIF can then be used. Thus, a functional relationships between PDI and q could probably be most readily constructed by regression analysis. The resulting function would be incorporated directly into a quality constraint in the form of Volume One's Equation (3), that is:

$$(PDI)^* = h(q) \tag{1}$$

where "*" denotes a parametric value for the quality index subject to which cost minimization must take place.

If the analytical derivation of a cost function were considered infeasible or undesirable, then the fact that index number values would have been recorded along with corresponding output flows at various sites should make these data especially amenable to "direct" estimation of a cost function by means, for example, of regression analyses. Volume One's cost function schematic, namely

$$C_{LR} = \varphi \text{ (q; input prices; } I^{*}) \tag{2}$$

might be rendered in regression form as

$$C_{LR} = \alpha_0 q^2 + \alpha_1 q + \alpha_2 w + \alpha_3 \cdot (PDI) \tag{2'}$$

where w represents input prices, and PDI denotes, again for illustrative purposes, the Mitre Corporation index. No attempt to implement an analysis as suggested by Equation (2') has been made in this project because the requisite "index-and-associated-output" observations at multiple treatment sites were not available for any of the three indexes reviewed in Volume One. This discussion should make clear, however, what kinds of observations would be needed and how they would be used to help make a summary index play the role of a QIF in the Classical Economics Model.

B. LINEAR PROGRAMMING MODEL (LPM)

In Section IV-B of Volume One, a linear programming model (LPM) was delineated, the central purpose of which was to show how "effective supply" facing a water user can be modified by (internal) user response to changes in quality-related parameters in the LPM. Thus, for example, the "upgrading" feature of the model allows one to evaluate how a water user would adjust his output and water input flows in the event that treatment costs of transforming relatively low quality water into usable higher quality were to change. Because other features of the model, as well as specifics on its empirical implementation, were presented in some detail in Volume One, this information will not be repeated here. However, a brief statement about the numerical example which is presented subsequently in in Section II-E is in order.

From available published data, it was not possible to ascertain sample values for enough of the LPM parameters to permit a meaningful "actual data" illustration. One missing datum, for instance, was information on endowment quantities by quality class (the E_i parameters of Inequalities (15) in Volume One). In addition, "production process" observations that would yield values for the needed recovery-for-reuse (g^j_{ik}) and input-output parameters (h_{ij}) were lacking. On the "positive" side, however, was the indication that needed cost data (especially that for upgrading) could be adapted from sources like Koenig (6) or Linstedt, et al. (7), provided that definitions of "quality" were couched in terms of specific impairments. This latter point, however, posed a data problem itself.

For these reasons of data incompleteness, therefore, it was decided that the LPM's operationality could best be demonstrated by using consistent dummy data which reflect parameter values of reasonable magnitudes relative to each other. This then is what has been used in the sensitivity analyses discussed in Section II-E. Following that numerical representation, there will be a brief reiteration of Volume One's more detailed specification of the kind of data water planners would need to obtain in order to apply LPM to an actual case.

II. ILLUSTRATIVE EXAMPLES

This chapter presents the numerical results of applying the CEM and LPM methodologies developed in Volume One. Sections II-A, B, C give successively more generalized versions of CEM, although only II-A is "complete" in that it takes its analysis all the way through the derivation of numerical total and marginal cost functions. The format in each of these sections is to show the full model being examined along with the essential steps in the mathematical derivation. For II-A, one important additional detail is relegated to Volume Two's Appendix. Section II-D discusses briefly some of the methodology for direct estimation of cost functions, and II-E presents a dummy data version of the LPM.

A. SINGLE-IMPAIRMENT CEM: MULTIPLICATIVE-ADDITIVE PRODUCTION FUNCTION

In an attempt to demonstrate the use of CEM in a meaningful but readily understandable way, a model has been formulated in which the quality constraint depicts only a single water impairment (turbidity) and the production function used, although hypothetical, is mathematically manageable and embodies intuitively expected properties of water treatment processes.

Delineation of the Model

Using the format in Volume One, the model can be stated formally as given below, where dimensions on the variables have been dictated largely by foreknowledge of the data that will be used for numerical implementation.

Thus, it is desired to minimize daily costs (with respect to decision variables x_1, x_2, and F)

$$C = w_1 x_1 + w_2 x_2 + rF \tag{3}$$

subject to

$$q^* = a_1 x_1 x_2 + a_2 x_2 F \quad \text{(output-production function constraint)} \tag{4}$$

$$\frac{nx_1 - (x_2/m)}{q^*} = s \quad \text{(quality constraint)} \tag{5}$$

where:2/

x_1 = millions of gallons of raw water processed per day

x_2 = pounds of coagulant used per day

F = dollar value of treatment facility capital investment

n = pounds of turbidity present, on the average, per million gallons of raw water

m = pounds of coagulant needed, on the average, to treat (remove) a pound of turbidity

q^* = millions of gallons of "treated water" output per day (q^* will become "any" output level q)

w_1 = procurement cost of raw water (as dollars per million gallons)

w_2 = cents per pound coagulant cost

r = daily capital charge

s = maximum pounds of turbidity per million gallons of treated water that can be tolerated by a user class, i.e., the maximum post-treatment turbidity concentration that is acceptable to a particular type of user (e.g., public water supply), as specified by a source like NTAC (9)

a_1, a_2 = production function parameters.

Thus, the model purports to find the input values that will minimize the cost of producing a specified amount of treated water, where the residual

2/ Sections II-C and IV-A of Volume One give more detailed descriptions of, respectively, the variables and this form of quality constraint, and should be referred to at this point if necessary. It should be recalled that the derivation to be presented uses plausible functional forms, but its chief purpose is merely methodology illustration.

turbidity concentration in the treated water is stipulated to be a certain level.

Meant to be a "black box" within which contaminated raw water, coagulant and capital inputs are combined to produce "treated" water, the relatively simple (but manageable) production function in Equation (4) depicts interaction ("input substitutability" in economics terminology) between raw water and coagulant and between coagulant and capital, while raw water and capital affect each other only through their relations with coagulant in the treatment process. In other words, if the amount of coagulant entering the black box were decreased while the raw water flow increased, then a trade-off effect could be realized in which the chemical would no longer be as "efficient" (it is "spread too thin") in removing pollutants, giving as a net result a status quo amount of treated water output, where the latter has implicitly both quality and quantity dimensions.[3/] Similarly, capital and coagulant substitute for each other in the sense that more capital can signify better technology and, therefore, less coagulant would be needed to help produce a given amount of treated water. The substitutability property is captured in the multiplicative aspects of the function, while the independence

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3/ For cost function derivation, quality aspects of the treated water output are accounted for explicitly in the quality constraint of Equation (5). In this connection, it should be noted that (5) represents a generalization of the quality constraint as given in Volume One because it depicts an explicit interrelationship among all the variables critical to the constraint, rather than just a monitoring function applied to the output. This more general form could easily be incorporated into the initial CEM by replacing Volume One's Equation (3) with an implicit function relation:

$$H(I^*; q^*; x_1, x_2, F) = 0.$$

This formulation imposes no specific form but merely denotes that a constraining relation among inputs, output and quality parameter does exist. The conceptual results of Volume One still hold.

characteristic is embodied in the additivity feature of the function. For this reason, Equation (4) can be referred to as a "multiplicative-additive" ("M-A") function.[4/]

Another feature of (4) is that it is homogeneous of degree two. Translated into economics, this means increasing returns to scale in production are presumed, and, while the degree implied here may or may not exactly represent the nature of water treatment processes, there is strong evidence in the literature[5/] to support an allegation that economies of scale do prevail in a long-run function point of view. Equation (4) is a reasonable representation of this fact.

In summary, then, although the M-A function was chosen primarily for its mathematical properties, it does evidence some intuitively meaningful traits. Our intention is not to advocate M-A as "the best" black box representation of water treatment, but it is encouraging to know that it does possess realistic properties.

Complete CEM Solution

Implementing the optimization procedure, the relevant Lagrangian function (cf. Equation (4) of Volume One) is

$$L = (w_1 x_1 + w_2 x_2 + rF) + \lambda[q^* - a_1 x_1 x_2 - a_2 x_2 F] + \theta[nx_1 - (x_2/m) - sq^*] , \tag{6}$$

where q^* denotes "any" feasible output level.

Setting first partial derivatives of L with respect to x_1, x_2, F, λ and θ equal to zero gives the "first order" optimization equations:

4/ Equation (4) is a specific case of a more general class of functions suitable for production function interpretation as discussed in Heady and Dillon (2), chapter 3. For the present instance of demonstrating cost function derivation, however, this M-A function was chosen mainly for its mathematical manageability.

5/ See, for example, Hinomoto (4) and Wollman and Bonem (12) as cited in Volume One.

$$(w_1 + \theta n) = \lambda a_1 x_2 \qquad (7)$$

$$(w_2 - \theta/m) = \lambda (a_1 x_1 + a_2 F) \qquad (8)$$

$$r = \lambda a_2 x_2 \qquad (9)$$

Equation (4) (10)

Equation (5). (11)

By successive substitution and algebraic manipulation,[6] the following solutions result (now writing q^* as any q):

$$x_1 = (sq + \sqrt{Nq}\,/m)/n \qquad (12)$$

$$x_2 = \sqrt{Nq} \qquad (13)$$

$$F = \frac{(n - a_1 N/m)\sqrt{q} - a_1 sq\sqrt{N}}{na_2\sqrt{N}} \qquad (14)$$

where $N = \dfrac{nr}{na_2 w_2 + (a_2 w_1 - a_1 r)/m.}$

By substituting these solutions in the cost equation of (3), costs are expressed solely in terms of output q, the quality parameter s, production function parameters a_1 and a_2, and input prices w_1, w_2, and r, as the conceptual analysis of Volume One indicated should be the case. Collecting terms algebraically and then incorporating abbreviated notation,[7] the (long-run) total cost function thus derived is

$$C = sAq + D\sqrt{q} \quad , \qquad (15)$$

while differentiating with respect to q gives the corresponding marginal cost function as

$$MC = sA + D/2\sqrt{q} \quad . \qquad (16)$$

6/ See Appendix for the "second order conditions" verification that a true minimum has been achieved.

7/ $A = (w_1 a_2 - ra_1)/na_1$

$D = \sqrt{N}\,(w_1 + mnw_2)/mn + r(mn - a_1 N)/mna_2\sqrt{N}$. Note also that C and MC are, respectively, Volume One's C_{LR} and MC.

Since s measures post-treatment residual turbidity, lower values for s signify more stringent quality standards. Thus, if the coefficient A is negative in (15) and (16), this means both C and MC increase as s falls; in other words more stringent water quality standards will be achieved only with higher total and marginal costs if $A < 0$.

Because of its relatively simple form, Equation (16) also enables straightforward determination of the incremental cost of an "incremental unit of quality." That is, the partial derivative of C with respect to s is

$$MC_s = \frac{\partial C}{\partial s} = Aq \tag{17}$$

and measures the change in costs due to a unit change in residual pounds of turbidity per million gallons of treated water output ("marginal pollutant alleviation cost"). Alternatively stated, Equation (17) calculates the cost of an additional unit of impairment, measured in terms of the cost of removing the impairment. In this case, it can be seen that these costs rise (linearly) as the level of output rises.

Parameter Estimation From Available Data

Turning now to an illustrative numerical estimation of these cost functions, it will be recalled that two tasks are involved: first, the a_1 and a_2 production function parameters must be ascertained; and then input price and treatment parameter values have to be specified. The former was done by estimating Equation (4)'s production function econometrically, while realistic representative values gave estimates of the latter parameters. Table II-1 summarizes the values deduced, while the discussion that follows gives further relevant details.

<u>TABLE II-1</u>

COST FUNCTION PARAMETER VALUES

Parameter	Symbol	Estimated Value	Dimensions	Source of Estimate[a]
Production Function	a_1 a_2	4×10^{-5} 3.939×10^{-9}	-- --	Regression on Koenig (6) Data[b]
Procurement Cost	w_1	2.07	\$/million gals.	Washington Suburban Sanitary Commission and Koenig (6)
Coagulant Cost	w_2	0.0322	\$/pound	Sample from Koenig
Daily Interest on Plant	r	2.13×10^{-4}	--	Sample from Koenig
Raw Water Parameter	n	346.11	lbs. of turbidity per million gals. raw water	Sample from Koenig
Treatment Parameter	m	0.70477	lbs. coagulant per lb. of turbidity	Sample from Koenig
Quality Parameter	s	2085; 1042.5; 83.4; 0	lbs of turbidity remaining per million gals. of treated water	Selected Values[c]

a/ See text for more detailed discussion.

b/ The regression-estimated production function was: $q = (4 \times 10^{-5})\, x_1 x_2 + (3.939 \times 10^{-9})\, x_2 F$, where values used for regresand and regressors are discussed in the text. F-statistic: $38.027 \gg 4.35 = F_{(1,20)}$ @ 95% confidence. Coefficient of Determination ("R^2"): 0.7918.

c/ These values (which correspond to, respectively, ppm ("parts per million") counts of 250, 125, 10, and 0) were chosen as representing a reasonable range for the parameterization analysis.

Thus, the production function regression used observations for 22 of the plants reported by Koenig (6). Explicit data for millions of gallons of treated water per day (q), pounds of coagulant used per million treated gallons, capital investment as cents per "gallons-per-day-of-capacity," and capacity in terms of millions-of-gallons-per-day are given for each of 22 treatment plants by Koenig (6). By combining these data, pounds of coagulant per day x_2 and investment F were readily deduced, where for the latter the (perhaps heroic) assumption has been made that the total plant (not just a portion) must be present to support the coagulant activity. Raw water input per day x_1 was not recorded separately, because Koenig took it to be identical to q. Ultimately that is what was done here (with due consideration for potential econometrics problems[8/]), but only after attempts to "define" x_1 explicitly went awry. That is, conceptually speaking, it would be desirable to have raw water couched in terms of the contaminants that make it "raw," and one of the regression attempts to derive an explicit functional relation between x_1 (measured as q) and turbidity levels was relatively successful.[9/] When the x_1 values thus generated were used in the production function estimation, however, a negative sign for a_1 resulted which contradicts economic intuition. For this reason and the fact that using Koenig's data directly serves very adequately for demonstrating the CEM methodology, it was decided to proceed with $x_1 = q$ as described in Footnote 8. To reiterate, though, the alternative form has been mentioned to point

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8/ Since regressing q on itself would force a unitary coefficient for $x_1 = q$ and zero for any other exogenous variables, care had to be taken not to use x_1 by itself as a regressor. This was accomplished due to the multiplicative parts of the M-A function.

9/ The form $x_1 = e^{\gamma} T^{\alpha}$, where T = pounds of turbidity, gave $R^2 = 0.8$ and significant regression coefficients.

out that it would be desirable if a water planner could have independent observations on x_1.

Table II-1 indicates that the regression analysis resulted in a relatively good fit and that, as a whole function, the F-statistic implies that a highly significant relation was achieved, even though it was due almost entirely to the "x_2F" term.[10/] It will be seen in Section II-B that different functional forms can result in more "balanced" statistical significance, even though other complexities may be introduced.

The parameters besides a_1 and a_2 listed in Table II-1 were evaluated by direct computation from recorded data. Expanding on the comments in the table:

w_1 was estimated by applying information on the ratio of procurement costs net of debt service to treatment costs (both as cents per 1,000 gallons) for 1964 (the reference date for Koenig's cost data) as provided by the Washington Suburban Sanitation Commission (WSSC).[11/] Because data for procurement costs net of debt service[12/] proved to be unavailable in the surveyed literature, it was felt that for the illustration purpose desired here, a "ballpark" figure would be adequate. Accordingly, WSSC realized a cost ratio of (procurement)/(treatment) = 0.264. With "treatment" characterized by coagulation in the present example (a simplification which should make the resultant estimate an overstatement), the estimate was calculated as

$$w_1 = (0.264)\cdot(\$/\text{lb. coagulant})\cdot(\text{lb. coagulant/mil. gal.})$$

where the two verbally-stated factors were taken as mean values of 18 comparable plants in Koenig's study.

w_2 is a representative value, taken as the average of Koenig's sample mean and the same mean computed by excluding four plants not in the regressions.

10/ The a_1 coefficient estimate was very insignificant (t-value of 0.5, as compared to 5.1 for the a_2 estimator), but it must be retained because it appears explicitly in the algebraically-derived cost function. That is, econometrics has not been employed as a "search" tool in this methodological example; the desired production function form has already been specified. Estimation is thus used to quantify two positive parameters which must both be used, regardless of statistical significance.

11/ Mr. Henry Benson, Assistant Treasurer of the WSSC, provided the relevant figures for WSSC experience; his help is gratefully acknowledged.

12/ Because of CEM accounts for capital costs explicitly, they should not also be in procurement costs.

r comes from Koenig's "fixed annual cost," computed, in part, as the annuity that 1 will buy for 30 years at 4%. From any standard annuity table, this value is 0.05783 which, when divided by 365 days per year, gives 0.000158 as a representative daily capital charge on a plant's original investment. Koenig then adds an additional 2% to the annual figure to cover taxes and insurance, giving a daily figure for that part of 0.000055. Adding the two components, the representative daily total capital charge is r = 0.000213. Although he notes that most of his plants were municipal plants and "...did not show costs for either taxes or plant insurance," the 2% is added to "...make the total costs more comparable between municipal and private operation."[13/]

n represents the sample average (for those plants reporting the information) turbidity content of raw water intakes computed as 41.5 ppm (parts per million) and then translated into pounds per million gallons by using the conversion factor 1 ppm = 8.34 lbs/mil. gal.[14/] to get n = (41.5)(8.34) = 346.11 lbs. per mil. gal.

m was computed as

(lbs. of coagulant per mil. gals.)/n ,

where the numerator was the sample mean for the same plants used for n.

s values were merely chosen to cover the range from 0 residual turbidity (assuming physically possible) to 250 ppm, the maximum raw water concentration in Koenig's sample. (The relation cited above was used to convert ppm into pounds per million gallons.) It is of interest to note that NTAC (9, p. 20) stipulates "virtually absent" as the desirable criterion for turbidity level in public water supplies, so this most stringent case has been covered in the example.

13/ Koenig (6), p. 299.

14/ Fair, Geyer, and Okum (1), p. 691. Strictly speaking, this conversion formula should be modified to account for temperature (and therefore density) differences, but for empirical operationality and purposes of methodology demonstration, the factor given by Fair, et. al., is totally workable. (Between 30°F and 86°F, water density varies with temperature only at the third decimal place, but the presence of foreign matter will affect this relation more.)

Derived and Enumerated Cost Functions

If the parameter values from the previous subsection are inserted into Equations (15) and (16)'s total and marginal cost functions, it is seen that the coefficient A is indeed negative, giving the previously-described "higher quality costs more" characteristic. Table II-2 gives the functions for the four selected values of s , along with the cost values for outputs ranging from 0.5 to 14 million gallons per day of output flow (q). In addition, the two extreme cases (250 and 0 ppm) are graphed in Figures II-1 (total costs) and II-2 (marginal costs). These exhibits show that for this illustrative example, the variation in total and marginal costs over the depicted output range is noticeable, while the cost differences between the different quality parameter (s) cases is of smaller magnitude but nonetheless is measurable.

Although there was nothing in the surveyed literature which would enable strict comparison with the values in Table II-2, a very crude check on order-of-magnitude can be obtained by reviewing results from Robert Smith (11). His Figure 8 records (in 1967 dollars) advanced wastewater treatment costs (operation, maintenance, debt service) vs. mgd design capacity for solids removal by coagulation and sedimentation. Because Smith's figures are for total processing costs (not just coagulation), one should probably expect them to be somewhat higher than the costs in our example. But, the fact that his numbers refer to advanced wastewater treatment (treatment applied to secondary effluent which has already been partially clarified) would, from a comparison viewpoint, act to offset the fact that Table II-2's values are based on Koenig's water purification (as opposed to waste treatment) data. These latter comments notwithstanding, however, Table II-3 presents Smith's

TABLE II-2

DERIVED TOTAL (C) AND MARGINAL (MC) COST FUNCTIONS[a/] AND VALUES[b/]

	$C = sAq + D\sqrt{q}$ and $MC = sA + D/2\sqrt{q}$							
	s = 2085 (250 ppm)		s = 1042.5 (125 ppm)		s = 83.4 (10 ppm)		s = 0 (0 ppm)	
Functions:								
C =	$-0.563q + 83\sqrt{q}$		$-0.282q + 83\sqrt{q}$		$-0.023q + 83\sqrt{q}$		$83\sqrt{q}$	
MC =	$-0.563 + 41.5/\sqrt{q}$		$-0.282 + 41.5/\sqrt{q}$		$-0.023 + 41.5/\sqrt{q}$		$41.5/\sqrt{q}$	
Output Range[c/] q =	0.5	14	0.5	14	0.5	14	0.5	14
Values For:								
C =	58.41	302.62	58.55	306.56	58.68	310.18	58.69	310.50
MC =	58.13	10.53	58.41	10.81	58.67	11.07	58.69	11.09

a/ Applying the data to the formulas in Footnote 7 gives A = -0.00027; D = 83.

b/ Total cost units are "$ per day;" marginal costs are "$ per million gallons per day."

c/ q values are 0.5 and 14 million gallons per day ("mgd").

FIGURE II-1

TOTAL COST FUNCTIONS

$C = sAq + D\sqrt{q}$

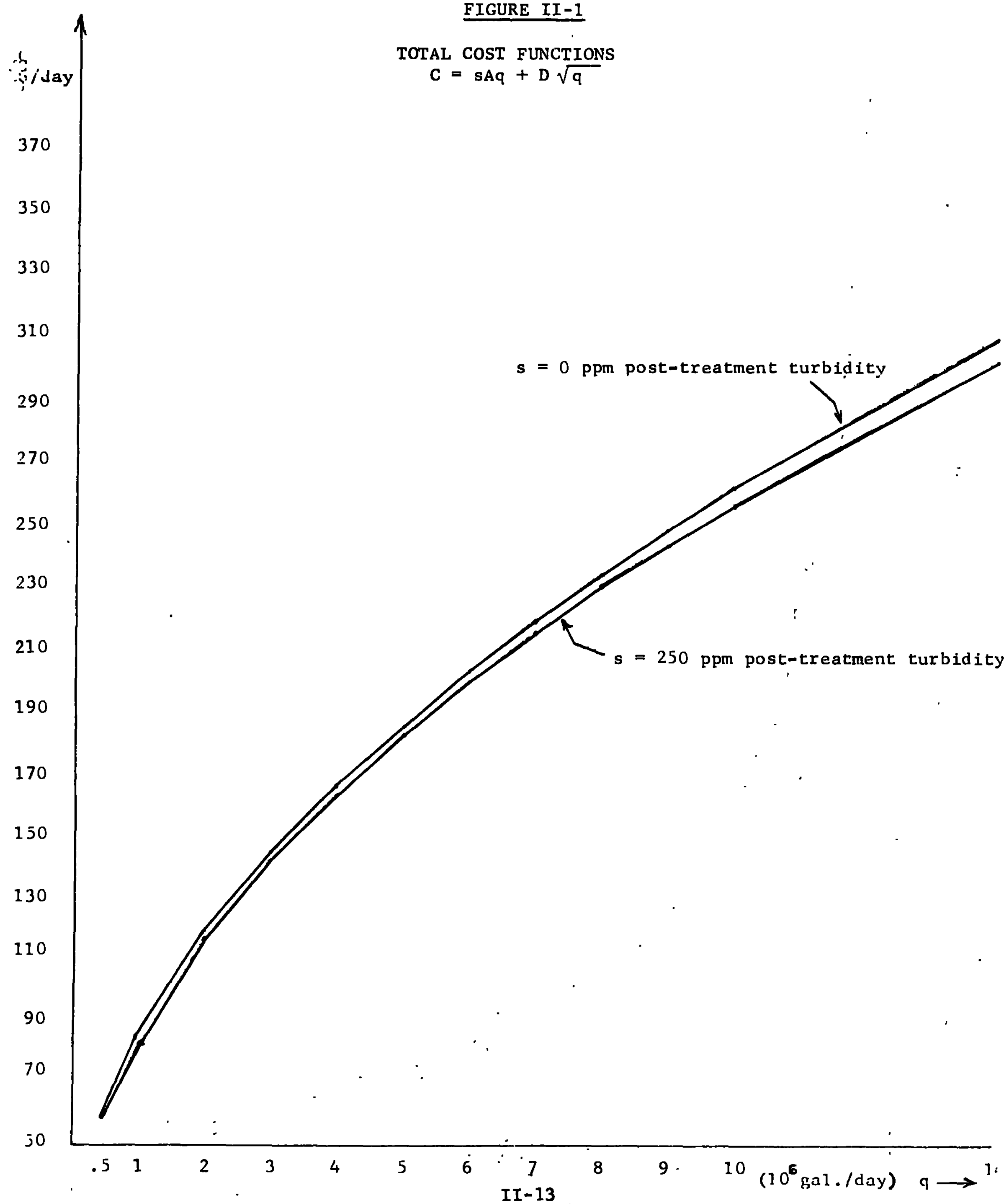

FIGURE II-2

MARGINAL COST FUNCTIONS

$MC = sA + D/2\sqrt{q}$

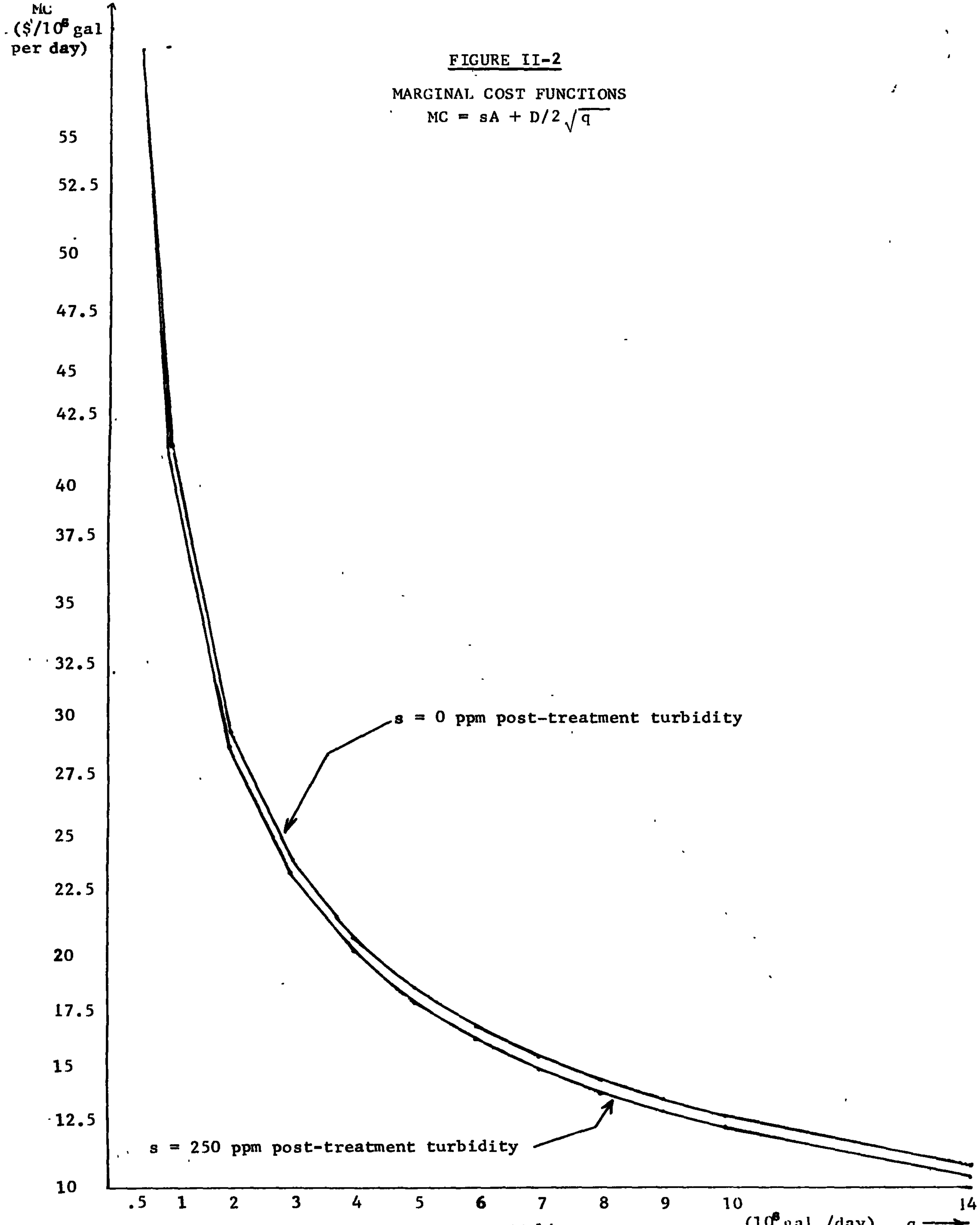

TABLE II-3

ADVANCED WASTEWATER TREATMENT COAGULATION-SEDIMENTATION COSTS[a/] AND COMPUTED CEM COSTS AT SIMILAR OUTPUTS[b/]

Design Capacity ζ (mgd)	Average Cost ($/1000 gals.)	Total Cost to Treat $\zeta \cdot 1000$ mgd	CEM Cost at $q = \zeta$ for $s = 0$
5	$0.039	$195	$186
10	0.037	370	262
14	0.036	504	310

a/ Columns 1 and 2 were read from "Total Treatment Cost" graph in Smith(11), Figure 8, p. 1552. Column 3 was computed by assuming plant operation at the design capacity.

b/ Column 4 was computed from the cost function for $s = 0$ in Table II-2, taking q = the design capacity ζ.

numbers and the "corresponding" costs from the CEM, but no strict comparison exists. All that can be noted is that the CEM example has generated total cost values which appear to be of reasonable "ballpark" order-of-magnitude--which is an encouraging vote for the methodology being tested.

Computed Marginal Pollutant Alleviation Cost (MC_s)

As a final point to consider, it is of interest to recall that the CEM cost function enables computing incremental costs of achieving additional quality (refer to Equation (17) and accompanying discussion). It was noted that, for the derived example, MC_s rises with output, a fact verified by the observation that Figure II-1's total cost curves diverge as q increases (Volume One, recall, pointed out that MC_s is reflected by the vertical distance between total cost curves). This could be interpreted as reflecting an intuitive notion that, although scale economies (cited earlier) with respect to daily flow and constant quality level exist, such is not the case across quality levels as daily flow increases. Whether this notion is in fact true is a matter of conjecture; the only means for readily testing it is the present analysis itself.

Equation (17), which is linear in its parameter,[15] can be used to obtain sample values for selected output levels:

q(mgd)	MC_s [$ per pound of turbidity removed per mgd]
0.5	-$0.000135
5	-$0.00135
10	-$0.0027

Hence, in the derived formulation, a reduction in the turbidity per mgd remaining after treatment has been applied costs an additional 0.0135 cents per pound of concentration if a plant treats ½ mgd, but it costs about an additional ¼ cents per pound if 10 mgd are treated. In other words, the

15/ $MC_s = \frac{\partial C}{\partial s} = Aq = -0.00027q$. Note again that mgd = millions of gallons per day.

incremental "cost" of turbidity pollution is about ¼ cents per pound per million gallons, measured at a treatment rate of 10 mgd.

Thus, it is seen that the MC_s concept can be a particularly useful water planner's tool as it enables a more explicit association of costs with impairments. In the following section, it will be noted that MC_s can facilitate analytic determination of solutions otherwise considered algebraically unmanageable.

In concluding this section, a summary is in order. Accordingly, it was seen that the CEM can be a workable concept; with appropriate data, a production function was estimated, and an analytically-derived cost function was numerically evaluated. The principal feature of the cost function was that a quality parameter appeared in it explicitly, making it possible to associate costs with specific levels of quality. Demonstrating the feasibility of such a result was a major purpose of this study--it has been accomplished.

B. SINGLE IMPAIRMENT CEM: COBB-DOUGLAS PRODUCTION FUNCTION

In this and the following section, two means of generalizing Section II-A's single-impairment CEM are explored. The first method is that of introducing a more general form of production function. Toward this end, therefore, consider the familiar Cobb-Douglas (C-D) function, used frequently in economic analysis. A "general multiplicative" function, the form relevant to the CEM is:

$$q = z x_1^a \, x_2^b \, F^c \tag{18}$$

where the input variables are the same as those in Part II-A, and the parameters requiring numerical specification are z, a, b, and c.

Estimated Production Function

As was noted in Volume One, the standard procedure for estimating an exponential function like (18) by means of regression analysis is to "linearize" it by writing it in (natural) logarithmic form:

$$\ln q = \ln z + a \ln x_1 + b \ln x_2 + c \ln F. \qquad (18')$$

Using Koenig's data in the form applied to the M-A function,[16/] the following regression results were obtained:

Parameter	Estimate	t-statistic	Overall Traits
z	1.26×10^{-5}	---	$R^2 = 0.91716$
a=b	0.2532	4.55	F-statistic =
c	0.7648	4.246	105.183

It is clear that the C-D function gives, in a statistical sense, a good "black box" representation of the treatment process in simplified form. Not only is a good (high R^2) and significant (F-statistic $\gg 8.10$, at 1% significance) fit achieved, but also each of the two regressor variables is itself a statistically-significant contributor (t-statistics are nearly equal and >2.53, at 1% significance) to the result. Furthermore, the exponent sum (which measures returns-to-scale for a C-D function) is $(a + b + c) = 1.2712$ which, because it exceeds 1, indicates increasing returns-to-scale, again supporting on this point the intuition that aided in structuring the M-A function. Econometrically speaking, therefore, there is substantial reason to at least examine functions of the C-D form when attempting to characterize a water treatment process.

16/ As noted previously, the available data forced measuring x_1 and x_2 as, respectively, q and ϵq, where ϵ = lbs. of coagulant per million gallons. The consequence of this was to imply a regression based on

$$q = z(\epsilon q^2)^{\gamma} F^{c}$$

to give estimates of only z, γ, and c. For the original (desired) function of Equation (18), therefore, one compares exponents to deduce that $a = b = \gamma$ since one can write $q = z(q)^{\gamma}(\epsilon q)^{\gamma} F^{c}$ as comparable to Equation (18).

Input Solutions--Methodology

In spite of statistical "success" at estimating its parameters, however, the C-D function proved to be unmanageable as regards enabling a complete analytic derivation of a cost function.[17] Accordingly, the analysis of this section, unlike that of Section II-A, cannot be carried to a final conclusion. It is nonetheless instructive to look at the CEM in terms of the C-D function to see where the derivational problem arises and to identify methodology that might be potentially useful in helping to resolve the problem.

Turning to a formal specification of the model, reference can be made to Section II-A's Equations (3)-(5), the only change required being the right-hand side of Equation (4). Thus, it is desired to minimize costs (with respect to x_1, x_2, F)

$$C = w_1 x_1 + w_2 x_2 + rF \tag{19}$$

subject to

$$q^* = z x_1^a x_2^b F^c \quad \text{(parameters } z, a, b, c, \text{ are presumed known)} \tag{20}$$

and

$$n x_1 - x_2/m = sq^*. \tag{21}$$

The Lagrangian function is

$$V = w_1 x_1 + w_2 x_2 + rF + \lambda[q^* - z x_1^a x_2^b F^c] + \theta[n x_1 - x_2/m - sq^*] \tag{22}$$

and the first-order optimizing equations to be solved are [cf. Equations (7)-(11)]:

$$(w + \theta n) - \lambda a q^*/x_1 = 0 \tag{23}$$

$$(w_2 - \theta/m) - \lambda b q^*/x_2 = 0 \tag{24}$$

$$r - \lambda c q^*/F = 0 \tag{25}$$

[17] This might not be totally unexpected, for even the "textbook" cost function derivation for a 2-factor C-D function requires considerable algebraic manipulation (see Henderson and Quandt (2), p. 85, for example), and only one constraint is involved there, not two as is the case for CEM.

Equation (20) (26)

Equation (21). (27)

Because of their non-linearity (especially Equation (20)'s restatement of the production function itself), these equations defy simultaneous analytic solution for all five variables needed (x_1, x_2, F, λ, and $\dot{\theta}$). It is here, therefore, that attempts to derive an exact cost function faltered.

Various approximation techniques could be employed at this point, probably the most efficient of which is described below. Because the degree to which any such method will be successful depends to a significant extent on what data are available, the lack of one such datum precluded numerical implementation in the present case. However, the approach one would use is outlined.

Summarizing key parts of the proposed methodology, first reduce three equations to only two and simultaneously eliminate one variable (λ) by solving (25) for λ and substituting for λ in (23) and (24). The resulting set of four equations in four variables is:

$$x_1/F = ra/c(w_1 + \theta n) \quad (23')$$

$$x_2/F = rb/c(w_2 - \theta/m) \quad (24')$$

Equations (20) and (21) are intact.

The object now is to:

(1) Use any three of these latter four equations to "solve" for x_1, x_2 and F in terms of the known parameters and θ

(2) Insert these "solutions" into the yet unused fourth equation, rendering it an equation in the single variable θ

(3) Solve the equation of (2) for θ

(4) Backtrack to (1) in order to insert the value of θ in the x_1, x_2, F formulas, making those variables now determinate.

In their stated form, these steps constitute a procedure for determining <u>exact</u> (not approximate) solutions; the approximative trait is introduced only when any one of the four steps cannot be carried out precisely.

Applying the above procedure to the case at hand (but omitting tedious details), the results can be listed corresponding to step number:

(1) Extensive algebraic manipulation of (23'), (24'), and (20) gives

$$\left.\begin{aligned} x_1 &= R_1^{(b+c)/S} \; R_2^{-b/S} \; (q^*/z)^{1/S} \\ x_2 &= R_1^{-a/S} \; R_2^{(a+c)/S} \; (q^*/z)^{1/S} \\ F &= R_1^{-a/S} \; R_2^{-b/S} \; (q^*/z)^{1/S} \end{aligned}\right\} \qquad (28)$$

where S

$S = (a + b + c);$

R_1 = right-hand side of Equation (23')

R_2 = right-hand side of Equation (24').

Because the R_1 and R_2 terms contain θ, the input solutions are indeed given in terms of various parameters of the CEM, and θ.

(2) Inserting the input solutions into the yet unused quality constraint (21) leads, after simplification, to the relation

$$q^* = ([nR_1 - R_2/m]/S)^{S/(S-1)} \cdot (zR_1^a R_2^b)^{1/(1-S)} \qquad (29)$$

which, through R_1 and R_2, contains only the variable θ.

(3)[18] Conceptually Equation (29), being a single equation in one unknown, has a solution for θ as a function of q^* and the other known CEM parameters. In fact, however, (29) is so highly non-linear that it is impossible to obtain an exact solution, and it is here, therefore, that approximation methods would have to be used.[19]

18/ By default, Step (4) is included here since approximation is introduced with attendant discussion about final input solutions.

19/ Because of its very irregular form, even standard computer programs for finding polynomial roots cannot be used.

Probably the most plausible approach would be to ascertain a feasible set of values for θ, and then use (29) to compute the q^* value corresponding to each θ. This (hopefully monotonic) correspondence relation between q^* and θ thus established, it would be recorded for later use in "reverse" order. That is, in ultimately plotting the cost function, values of q^* (or, simply "any" q) would first be specified, and these, through the relation here discussed, would determine values of θ; the latter would then determine values of the inputs by means of Equations (28). In the manner then of Section II-A, the input values would be inserted into cost Equation (19), giving the desired plot of cost vs. q.

Critical to implementing such an approach, however, is the independent "guess" at a feasible range of θ values. Because of data limitations in the present example, attempts at ascertaining such a range were unsuccessful. Nonetheless, a practical method for facilitating empirical determination of a θ-range was developed. Presented in the next subsection, it indicates what kind of data observations would be needed.

<u>Use of the MC_s Concept as an Implementation Aid</u>

In Chapter I (and again in Section II-A), the notion of marginal pollutant alleviation cost (MC_s) was presented in an "ex post" sense; that is, within the framework of already-derived cost functions. It can be shown, though, that the concept has meaning at an earlier stage of the cost function analysis and therefore may be potentially useful for facilitating empirical derivation of cost functions. To be specific, the Lagrange multiplier θ associated with the quality constraint is directly related to MC_s by

$$MC_s = \frac{\partial C}{\partial s} = -\theta q^* \qquad (30)$$

where the value of θ implied is that at the minimum cost optimum. That some kind of relation exists is a well-known result for constrained optimization analysis,[20] but it is informative to describe briefly how the specific form in Equation (30) is deduced.

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[20] See, for example, Intriligator (5), pp. 36-38.

Recall, therefore, the Lagrangian function of Equation (22) and visualize it evaluated at the minimum cost optimum. This means that the decision variables x_1, x_2, F, λ, and θ are taken to be in their optimal solution forms wherein they are functions of all the CEM parameters, including s. Partial derivatives of these variables with respect to s are thus well-defined, so the partial of optimal V with respect to s can be computed and written, after collecting terms, as:

$$\frac{\partial V}{\partial s} = V_1 x_1^s + V_2 x_2^s + V_F F^s + V_\lambda \lambda^s + V_\theta \theta^s - \theta q^* \qquad (31)$$

where the symbols V_1, V_2, and V_F denote partials of optimal V with respect to the three inputs; V_λ and V_θ denote constraint Equations (20) and (21), respectively; and the "s" superscripts denote partial derivatives of each variable (evaluated at the optimum) with respect to s. The presumption of optimality means that the five "V_i" factors are simply the first-order optimization Equations (23)-(27) and therefore equal to zero. In addition, because constraints (20) and (21) must be satisfied at the optimum, the terms representing the constraints disappear from Equation (22), leaving optimum V simply equal to optimum (minimum) cost C. Incorporating these results into Equation (31) causes the first five terms on the right-hand side to vanish and enables replacing "V" by "C" on the left-hand side; in short, Equation (31) becomes Equation (30). Transposing therefore shows that $\theta = -MC_s/q^*$.

Stated verbally, the optimum value of the Lagrange multiplier associated with CEM's quality constraint is simply the negative ratio of marginal pollutant alleviation cost (at output level q^*) to the output level itself.[21/] In other words, if one can ascertain an independent estimate

21/ Since MC_s itself is negative, θ can be expected to be positive. Note that the sort of analysis demonstrated here is, in economists' terminology, an example of determining "shadow prices," namely the "cost" (in terms of effect on the value of an objective function) of changing a constraint by one unit.

for MC_s at a specified output level, then the corresponding value of θ can also be readily estimated. This initial estimate (call it θ_o) could then be used to implement the approximation procedure of the previous subsection as follows:

- Insert (through R_1 and R_2) θ_o into Equation (29), and compute the right hand side, RHS(θ_o)
- Compare the value of RHS with the q^* value that gave rise to θ_o
- If $q^* = \text{RHS}(\theta_o)$, then Equation (29) is satisfied and θ_o is optimal for the given q^* and other CEM parameters specified in (29); θ_o can then be substituted into Equations (28) to obtain input solutions corresponding to the given q^* value, and from these a cost value $C(q^*)$ can be computed
- If $q^* \neq \text{RHS}(\theta_o)$, then different values of θ would need to be tried by successive iteration until convergence of RHS(θ) to q^* could be achieved; the convergence θ value would then play the role of θ_o in the previous step.

If the entire procedure is repeated for a range of feasible q values, then the locus of cost vs. q (for a specified s value) is determined; in short, a cost function will have been derived. If, in addition, the value of s in Equation (29) is changed, and all steps are repeated, then the cost function corresponding to the new s results. In other words, the desired family of cost curves corresponding to the C-D production function (cf. Figure II-1) will have been derived. That it was not in fact done for this study was due to the lack of an independent MC_s observation.

To reiterate in concluding this section, the central purpose here was to show that a more general (but still relatively simple) production function is compatible with the CEM example being studied, but some (potentially serious) complications as regards obtaining an ultimate solution are introduced. On the other hand, a methodology for overcoming the solution problems has been developed and appears workable provided an initial observation on one datum can be obtained.

C. THREE-IMPAIRMENTS CEM: COBB-DOUGLAS PRODUCTION FUNCTION

Another way to generalize the CEM analyses examined so far is to increase the number of impairments included. Implicitly this has already been done in Volume One's discussion (see Section IV-A) of the forms that the quality constraint can assume where it was noted that either "macro" or "micro" interpretations could be used, the former being particularly applicable to the case of an index number form of QIF which, by definition, accounts for multiple impairments. If, on the other hand, it is desired to have a separate rendering for each contaminant, then the only alternative is to fashion multiple quality constraints. This section addresses itself to that approach.

Formulation of Model

It is sufficiently general to consider three impairments; using Section II-A's symbols therefore as guides, define these comparable counterparts:

q = gallons per day of treated water output, as before

x_1 = gallons of raw water per day, as before

(j=2,3,4)

x_j = amount of Input j used daily

n_j = amount of Impairment j present per gallon of raw water

m_j = amount of Input j needed to alleviate a unit of Impairment j

F = dollar amount of invested capital

r_j ; w_i (i=1,2,3) = unit input prices

s_j (j=2,3,4) = quality parameter, namely, a specified post-treatment amount of Impairment j per gallon of q.

Using an expanded Cobb-Douglas production function for further generalization, the CEM becomes (decision variables are x_i's and F):

$$\text{Minimize } C = \sum_{i=1}^{4} w_i x_i + rF \tag{32}$$

subject to

$$q^* = z \cdot \prod_{i=1}^{4} x_i^{a_i} F^c \quad (z,\ a_i\text{'s},\ c \text{ are parameters}) \tag{33}$$

and

$$n_j x_1 - x_j/m_j = s_j q^* \quad \text{for } j = 2,3,4 \text{ (hence, 3 equations)} \tag{34}$$

(These equations are directly comparable to Section II-B's Equations (19), (20), and (21), respectively.)

The Lagrangian function is

$$W = \sum_{i=1}^{4} w_i x_i + rF + \lambda \left[q^* - z \cdot \prod_{i=1}^{4} x_i^{a_i} F^c \right] \tag{35}$$

$$+ \sum_{j=2}^{4} (n_j x_1 - x_j/m_j - s_j q^*) \cdot \theta_j$$

which shows that optimization occurs now with respect to nine total variables (five inputs and four Lagrange multipliers). Reviewing the first-order equations in the previous section will give clear indication that obtaining a solution to this multi-constraint version of the CEM is at best a formidable task. In fact, the likelihood of deriving an exact cost function is definitely more remote than in the single-impairment C-D function case. Nonetheless, a solution is conceptually feasible, and extension of the methodology described in Section II-B can facilitate finding it.

Solution Methodology

Following the pattern of the previous case, first-order equations are derived by setting first partials of W with respect to the nine variables equal to zero (denote these as $W_y = 0$, where y represents any of W's nine variables). Just as was the case there, here too it is possible to eliminate the variable λ by solving one of the five equations for which y is an input for λ and then replacing λ in the other four "y = input variable" equations. The result of this exercise is four equations of the form[22/]

$$x_i/F = B_i \qquad (i = 1,\ldots,4) \tag{36}$$

where

$$B_1 = ra_1/c(w_1 + \sum_{j=2}^{4} n_j\theta_j) \text{ and } B_i = ra_i/(w_i - \theta_i/m_i)c$$

for i = 2, 3, 4. The logarithmic transformations of (36) and (33) then constitute a system of five highly non-linear equations in the five input variables and the four θ_i's:

$$\left.\begin{aligned} &\ln(x_i) - \ln(F) = \ln(B_i) \quad (i = 1,\ldots,4) \\ &\sum_{i=1}^{4} a_i \ln(x_i) + c\ln(F) = \ln(q^*/z)\ . \end{aligned}\right\} \tag{37}$$

Equations (37) are solvable for each of the x_i's and F as functions of all the CEM parameters (except the s_i quality parameters) and the θ_i's, these being components of the B_i terms.[23/]

From here, one would proceed as before to derive results by iterative computation. The first step would be to substitute the input solution functions of θ_i's into the three (not-yet-used) quality constraint Equations (34) in short, as might intuitively be expected, the three-impairment analog of Section II-B's Equation (29) is three simultaneous equations relating q^*, s_i,

+

22/ Equations (36) were derived by solving first-order equation $W_F = 0$ for λ. Throughout this section, mathematical details have been held to a minimum to facilitate readability.

23/ The solutions are all of the general form

$$x_k = (q^*/z)^{1/S} \cdot \prod_{i=1}^{4} B_i^{-\alpha_i/S} \qquad (k = 1,\ldots,5;\ x_5 = F)$$

where $S = a_1+a_2+a_3+a_4+c$ and $\alpha_i = a_i$ for $i \neq k$, but $\alpha_i = a_i - S$ for i=k. It is seen, therefore, that these solutions have the same form as those for the previous case, given by Equations (28).

other CEM parameters and θ_i values. These constitute the conceptual basis for obtaining the θ_i solutions in terms of all CEM parameters. The inability to solve the relations exactly, however, leads to a consideration of the approach suggested earlier. Hence, a relation identical to Equation (30) can be derived for each of the three impairments ($MC_{s_i} = -\theta_i q^*$), which means that if initial estimates of the marginal pollutant alleviation costs (MC_{s_i} for i = 2,3,4) can be specified, then they can be used as starting values in an interative process seeking to make each of the three "right-hand side functions of θ_i's" of (34) equal to q^*. Simultaneous equality would signal optimal θ_i values, from which the solutions in Footnote 23 would give optimal inputs. From these the cost function value for q^* is determined. Repeating the procedure for alternative q^* values traces a C_{LR} curve for one set of s_i quality parameters, whereas changing any of the s_i's generates a family of curves just as before.

Concluding this section with a word about empirical operationality, it bears reiteration that, apart from the MC_{s_i} estimate (and even more fundamental), a multi-constraint generalization of CEM requires quantification of the various quality constraints. In the published data survey, the detail needed for numerical specification of several constraints did not exist. This is therefore an indication that, if water planners desire to implement the kinds of concepts developed here, a systematic means of assembling the requisite numerical observations will need to be devised.

D. COST FUNCTIONS BY DIRECT ESTIMATION--BRIEF COMMENT

To aid in retaining perspective, it is well to mention the alternative means of deducing cost functions. Because no actual estimates of this type were calculated, however, and because discussion of this topic has already been presented in Chapter I (in connection with Equation (2') there), the commentary here will necessarily be brief.

The common central point of the preceding three sections has been the economic theory that a production process underlies the ultimate determination of a cost function. It was the purpose of the production function-output constraint in each CEM example to convey this fact. Intuition would say, therefore, that different production functions (and hence different forms for the constraint) would give rise to different forms of cost functions. Such is, in fact, the case, being precisely what was observed when the M-A function of Section II-A led to a derivable cost function equation (what has been referred to in the text as an "exact," or "analytic," solution to the first-order equations), whereas II-B's CEM with a C-D function could not be solved analytically.[24] When an exact solution can be deduced, the result constitutes a very explicit demonstration of how production characteristics affect costs. Because similar discussion applies to the quality constraint, it is clearly desirable to derive a cost function if all possible. Only then can one be "sure" that the "correct" interaction of inputs and quality factors has been accounted for.

As was seen, however, it will not always be possible to obtain an exact solution. On the other hand, the "black box" representation of the production function that one chooses to work with may be too unrealistic in form, or may (if it is estimated econometrically) yield parameter estimates that are too unrealistic, to be useful. In any of these cases, an alternative means of associating costs with "known underlying" parameters must be used, and it is here that thoughts turn to regression analysis

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[24] By itself (i.e., without a quality constraint included), the C-D function <u>will</u> lead to a determinate cost function. The point here, though, is that even with a quality constraint, the M-A function gives a cost function, whereas the C-D will not.

and the approach suggested in Chapter I. It should be noted, however, that the use of a quality index as a regressor as shown in Equation (2') is not the only form amenable to effecting quality-inclusion. To the contrary, if the regression approach were used, it might prove very worthwhile to use traits associated with specific impairments (e.g., turbidity counts) as regressors, provided data observations can be obtained.

E. THE LINEAR PROGRAMMING MODEL (LPM)

It was noted in Chapter I that attempts to devise an "actual data" example of Volume One's LPM were unsuccessful. For this reason, dummy data of reasonable relative sizes have been generated as a means of testing the model to demonstrate its capability for registering the effect of parameter changes.

Because the description and formal presentation of the model were given in some detail in Section IV-B of Volume One, there will not be re-specification here. Accordingly, the reader should refer to Volume One for definitions of the variables as well as additional understanding and interpretation of the results given here.

Numerical Specification of the LPM

Briefly recalling that the LPM depicts profit maximization subject to technological, natural endowment, recovery-for-reuse, and input/output constraints, the most efficient way to present the illustrative example(s) is in "tableau" format, showing the coefficient values on each variable as it appears in both the inequality constraints and the objective function.[25/] Accordingly, each of the first 12 rows of Table II-4 records the coefficients of the LPM variables (first 14 columns) as they appear in one of the constraints, plus the "right-hand constant" for that constraint (either of the last two columns). The last two rows give objective function coefficients for the two different cases presented and discussed. A blank cell means the variable (column heading) has a value of zero in the use (row) indicated. It is critical to understand that all the constraints have been written so that all terms involving variables are on the left side of the inequality signs; only constants appear on the right side.[26/]

25/ In matrix form, a representative LP formulation is to maximize $u'x$, subject to $Ax \le d$, where u', x, and d are vectors of, respectively, objective function coefficients, unknowns, and "right-hand constants" from the constraints; and A is the matrix of constraint coefficients. The tableau given in Table II-4 is the array composed of:

$$
\begin{array}{c||c|c|c}
A & d_1 & d_2 & \cdots \\
\hline\hline
u_1' & & & \\
\hline
u_2' & & & \\
\hline
\vdots & & &
\end{array}
$$

The multiple entries u_i', d_i denote alternative choices (cases) of objective function coefficients and right-hand constants sets, respectively.

26/ As an example, the recovery constraint on Quality 2 water from Activity a (see Inequality (16) for i = 2, j = a in Volume One) is, in symbolic form:

$$-g_{e1}^{a} x'_{1a} - g_{e1}^{a} x_{12}^{a} - g_{e1}^{a} \tilde{x}_{1a} - g_{e2}^{a} x'_{2a} + (1-g_{e2}^{a})\,\tilde{x}_{2a} - g_{e2}^{a} y_{21}^{a} \le 0.$$

When the values $g_{e1}^{a} = 0.05$ and $g_{e2}^{a} = 0.02$ are inserted, then Row 6 of Table II-4 is readily established.

TABLE II-4

COEFFICIENT ARRAY FOR LP MODEL EXAMPLES

Constraints1/	Variables														≤ Right Hand Constants	
	q_a	q_b	x'_{1a}	$\bar{x}^a_{12}$	$\tilde{x}_{1a}$	x'_{2a}	$\tilde{x}_{2a}$	x'_{1b}	$\bar{x}^b_{12}$	$\tilde{x}_{1b}$	x'_{2b}	$\tilde{x}_{2b}$	y^a_{21}	y^b_{21}	Set 1	Set 2
Technological-	1														100	100
Inequalities (14)		1													150	150
Endowment-			1					1					1	1	13	8
Inequalities (15)				1		1			1		1				3000	15
Recovery-			-.03	-.03	.97	-.01	-.01						-.01		0	0
Inequalities (16)			-.05	-.05	-.05	-.02	.98						-.02		0	0
								-.01	-.01	.99	-.001	-.001		-.001	0	0
								-.02	-.02	-.02	-.005	.995		-.005	0	0
Input/Output-	1.1		-1	-1	-1										0	0
Inequalities (17)	1.5					-1	-1								0	0
		0.9						-1	-1	-1					0	0
		2.0									-1	-1		-1	0	0
Obj. Fn. Coeffs.2/ Case 1	100	100	-1	-10	-.5	-1	-.4	-1	-10	-2	-1	-1.5	-.005	-.005		
Case 2	100	150	-1	-10	-.5	-1	-.4	-1	-10	-2	-1	-1.5	-.005	-.005		

1/ Inequality numbers refer to Volume One, Section IV-B.

2/ For example, the entry -0.5 in Column 5 (for both cases) means that the coefficient of $\tilde{x}_{1a}$ (that is, $\tilde{c}_{1a}$) is -0.5 in the profit objective function of Volume One's Equation (13); similarly, output price p_b (Column 2) is 150 for the first objective function but is set at 100 for the second.

Although it has been noted that the data in Table II-4 are dummy values, it bears mention that the numbers do reflect two intuitively important considerations. First, the recovery constraint coefficients represent the fact that one would ordinarily not expect to recover from what went in as a particular quality of water more than the entering amount. That is, one expects water degradation with use to occur. Second, output price values p_a and p_b have been chosen so as to ensure *a priori* that profit can be positive[27/] (this guarantees an initial feasible solution). In addition, it can be pointed out that each of the two sets of right-hand constants reflects a relatively scarce natural endowment of high quality (Quality 1) water as compared to the lower quality input. Such relative magnitudes make it more likely that the model's upgrading feature will be exercised, which was felt to be a desirable part of an illustrative example.

Some Sample Results

Turning to the numerical examples themselves, Table II-5 gives results for the four cases which represent the possible combinations of Table II-4's objective function and right-hand constants' sets of values. Since actual data have not imposed specific dimensions on the results, they must be understood as simply "output units," "units of Quality 1 water recovered from Activity b," etc.

27/ Let c^m_{1a} and c^m_{2a} be the minima of the Activity a unit costs associated with Qualities 1 and 2 water, respectively, and let x_{ia} be a's total usage of Quality i water. Now, if $p_a < c^m_{1a}h_{1a} + c^m_{2a}h_{2a}$, then (multiply through by q_a) $p_a q_a < c^m_{1a}x_{1a} + c^m_{2a}x_{2a}$ follows since $x_{ia} \geq h_{ia}q_a$ by Volume One's Inequalities (17). In this last inequality, the left side is total net revenue for Activity a, while the right side is total water costs. Because revenue exceeds costs, profits are negative. It thus follows that p_a must be set so that the first inequality in this footnote is reversed if Activity a is to have a "chance" for non-negative profits.

TABLE II-5

OPTIMAL SOLUTIONS FOR FOUR LPM SCENARIOS

	q_a	q_b	x'_{1a}	$\bar{x}^a_{12}$	$\tilde{x}_{1a}$	x'_{2a}	$\tilde{x}_{2a}$	x'_{1b}	$\bar{x}^b_{12}$	$\tilde{x}_{1b}$	x'_{2b}	$\tilde{x}'_{2b}$	y^a_{21}	y^b_{21}	Joint Profit
C1-S1*/	100	150	13	92.2	4.8	141.5	8.5	0	133.35	1.65	300	0	0	0	22,280
C2-S1	100	150	0	105.2	4.8	141.5	8.5	13	120.35	1.65	300	0	0	0	29,781
C1-S2	0	8.04	0	0	0	0	0	7.15	0	0.088	15	0.225	0	.85	1,183.2
C2-S2	9.32	0	8	1.8	0.45	13.19	0.79	0	0	0	0	0	0	0	892.5

*/ The identification "Ci-Sj" denotes the LPM solution for Table II-4's Coefficient Case i combined with Right-Hand Constants Set j.

A comparison of Rows 1 and 2 shows how the model can record the effect of a change in relative magnitudes of output prices. Thus, with an increase in p_b from 100 (Row 1) to 150 (Row 2), it is seen that, although absolute output levels do not change, there are changes in usage of water input: direct procurement of Quality 1 water by Activity a falls by exactly the same amount that upgrading Quality 2 water rises, while just the opposite holds for Activity b. In other words, the higher price for b's output makes it worthwhile to obtain more Quality 1 water for b by direct procurement, even though the total amounts of each quality are the same for both cases. Joint profits rise significantly, reflecting simply the higher p_b value. It is further of interest to note that, as would be expected (since upgrading occurs), there is no diversion of Quality 1 water to Quality 2 uses in either of these cases.

Comparing Row 1 with Row 3 demonstrates how one can assess the effect of a change in endowments, since Row 3 depicts (primarily) a rather drastic reduction in available Quality 2 water. The consequence of this is seen to be a complete cessation of Activity a production (and, consequently, there is no usage of either grade of water), probably because the input/output relations cause it to be more profitable under these conditions to shift all available water into q_b. In addition, the non-zero value for y^b_{a1} shows that the short supply of Quality 2 water triggers diversion of Quality 1 water into Quality 2 uses (and, again, this is a mutually exclusive occurrence with upgrading water).

As a third "taste" of the LPM, move from Row 3 to Row 4, again depicting the rise in price p_b examined before, but now within the context of reduced natural endowments. The effect here is seen to be a "flip-flop" from q_b production exclusively to q_a exclusively, with a decline in joint profits. One plausible explanation for this is that the rise in p_b tries to call forth greater output, but the endowment constraints prevent such from occurring. Instead, all production shifts to q_a where recovery cost is lower. In addition, unlike q_b production (where some Quality 1 water was diverted), the "new" q_a production fully utilizes the available eight units of Quality 1 water, plus more obtained by upgrading. It is of interest to note that the drop in q_b with rise in p_b could be consistent with a highly elastic demand for Activity b's output.

The three pairwise comparisons reviewed above merely hint at the broad flexibility which characterizes the LPM as regards analyzing the effect(s) of parameter changes. Indeed, the number of parameter change combinations that one could hypothesize and evaluate for the two-quality two-output model presented here, considering only changes in objective function coefficients and/or constraint right-hand constants, equals 2^{168}--an enormous number! And the number grows even greater when (a) changes in "left-hand" constraint coefficients are allowed, and/or (b) the model is expanded to include more activities or water qualities. In short, the LPM permits a wide range of choice with respect to kinds of sensitivity analyses that can be evaluated.

As a final comment on the LPM, a brief restatement about empirical implementation can be made. Put succinctly, Table II-4 conveniently summarizes all of the parameters for which actual data observations would need to be obtained in order to apply the LPM to an actual situation. Thus, natural endowment quantities, recovery-for-reuse, input/output ratios, output prices and unit water input costs (procurement, upgrade, recovery, and diversion) would all have to be specified. Prior to collecting this kind of information, as was pointed out earlier, "quality" has to be defined so that appropriate measurements can be made. Referring back to the CEM examples, one possible approach to this issue would be to consider defining quality in terms of impairment concentrations, specifying different levels as the demarcation points between successive qualities. Requisite upgrade costs would then be the cost(s) of removing enough impairment to cause re-classification of an amount of water to a higher quality level.

III. MAJOR IMPLEMENTATION CONSIDERATIONS--SYNOPSIS

Although the illustrative examples of Chapter II can serve at least partly and implicitly as a "user's manual" for technique implementation, it is helpful to collect some of the central points into a more concise and systematic presentation. In this way, a practitioner can be alert, in an overview sort of way, to important aspects of applying the "supply side" concepts developed in the study. What follows, therefore, is a condensed recapitulation of topics with empirical relevance.

A. THE UNITS OF MEASUREMENT

Throughout Chapter II, it was stressed that the (desired) dimensions of specific variables in either the CEM or LPM are a critical consideration as regards either model's being empirically operational. In general, the type and form of available data will restrict the units of measurement that can be used, but these need to be checked for consistency with the model being anticipated for use. Volume One has shown, fortunately, that crucial parts of both CEM and LPM are amenable to broad interpretation. In the former, for instance, it was emphasized that the treatment variable x_2 can have "micro" (as illustrated in this volume) or "macro" dimensions. Similarly, CEM's quality constraint can be handled by either a (multiple) specific impairment constraint(s) or a summary index. For LPM, it has been noted that the output variables can be measured variously.

Common to both models is the need to fix the notion of how quality would be measured. In a sense, this is the crux and purpose of this entire study. For example, Section II-A's CEM was numerically implementable only because it was possible to convert ppm ("parts per million") observations on turbidity into at least a workable form of pounds per million gallons. Otherwise, the CEM quality constraint, which was formulated as post-treatment pounds per gallon residual impairment concentration, could not have been quantified. In addition, it was clear from the example that the dimensions of quality can affect in obvious ways how input prices must be measured.

In short, the central task of Volumes One and Two of this project has been to determine how quality factors can be embedded explicitly into water supply functions. Although it has been seen that general frameworks for approaching the problem could be established, the applicable specific form(s) will differ among different cases. This means that units of measurement must be determined separately for each distinct situation.

B. SUMMARY OF PURPOSES AND LIMITATIONS OF THE MODELS

This final section attempts to "draw things together" by summarizing key characteristics of each of the three methodologies presented. Because full discussion of each point has already been given in Volume One and the first two chapters of this volume, the statements are merely listed, without amplification.

Classical Economics Model (CEM):

- The purpose of CEM is to derive cost functions analytically which account explicitly for (a) an underlying production process, and (b) quality-associated parameters.

- The quality indicator function (QIF) is a general enough concept to permit liberal interpretation, but its dimensions must be specified precisely and be consistent with data that will be used.

- CEM's "black box" production function may not be amenable to engineering specification. In this event, estimation must be employed, with special attention paid to how variables are defined and measured.

- Complex forms of the quality constraint and/or production function will very likely preclude determining analytic solutions; in such cases, approximation techniques would have to be employed.

- CEM enables computation of marginal pollutant alleviation costs.

- Although CEM permits using general forms of production functions, any specific form selected needs to be checked for consistency with economic and engineering intuition about treatment process traits.

Hypothesized Cost Functions:

- Determining cost functions by estimating hypothesized forms is an alternative to analytic determination and, as such, does not account for underlying functional relations among costs, quality parameters, and production (treatment) processes.

- Regression estimates of hypothesized forms allow using either distinct impairment-specific quality variables or summary quality indexes, but either entity has to be observable along with corresponding output and cost observations.

Linear Programming Model (LPM):

- LPM deals with the question of quality in relation to supply by tracing the effect on productive activities' water usages due to changes in specified quality-related parameters.

- As formulated here, LPM does not permit associating costs with particular impairments.

- LPM requires an exogenous specification of distinct (measurable) quality classes of water. In other words, the inclusion of explicit quality parameters is external to the model itself.

- The LPM does not require determination of a relatively "complex" production function, but the input/output coefficients (which do require specification) impose a prescribed fixed-proportions production relation between outputs and water inputs.

- "Shadow price" calculations are possible with LPM, and these enable estimating the effect on profits due to relaxing a constraint by a unit amount. Thus, for example, the effect of increasing the endowment of low quality water can be deduced without re-solving the model parametrically.

- Documented linear programming algorithms are readily available to facilitate solving the LPM as well as dealing with large scale problems.

With the above enumeration, we conclude our study of supply aspects of water supply analysis. Briefly summarized, we have demonstrated that it is both conceptually possible and empirically feasible to incorporate explicit quality considerations into water supply functions. Methodologies have been specified and examples exhibited. What remains to be done is an integration with demand topics; this is the content of Volume Three.

IV. REFERENCE LIST/BIBLIOGRAPHY

Implicitly all citations in the bibliography to Volume One are also applicable to Volume Two. The specific items which have been referenced in the text, however, are listed below.

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MATHEMATICAL APPENDIX

SECOND-ORDER CONDITIONS FOR SINGLE-IMPAIRMENT CEM WITH M-A FUNCTION

This appendix sketches verification that Section II-A's solution is indeed a minimum cost solution, meaning that the cost function in Equation (15) is a true cost function. Analogous to the familiar calculus condition (for a single-variable function) that a positive second derivative evaluated at a stationary point ensures that the function has a local minimum there, similar sufficient conditions exist for multivariate function in which second partial derivatives are employed.[1/]

Applying the technique to the 3-variables and 2-constraints CEM in question, form the 5 x 5 "bordered Hessian determinant" whose elements are second partials of the Lagrangian function in Equation (6), evaluated at the solution point implied by first-order Equations (7) - (11). Using the double subscript form L_{ij} to denote the second partials,[2/] then symbolically the determinant is

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1/ See Intriligator (5, p. 35); Samuelson (10, p. 378); or Henderson and Quandt (3, p. 407) for details.

2/ Thus, for example, $L_{1F} = \frac{\partial L}{\partial F \partial x_1}$, and $L_{2\theta} = \frac{\partial L}{\partial \theta \partial x_2}$.

$$D_5 = \begin{vmatrix} L_{11} & L_{12} & L_{1F} & L_{1\lambda} & L_{1\theta} \\ L_{12} & L_{22} & L_{2F} & L_{2\lambda} & L_{2\theta} \\ L_{1F} & L_{2F} & L_{FF} & L_{F\lambda} & L_{F\theta} \\ L_{1\lambda} & L_{2\lambda} & L_{F\lambda} & 0 & 0 \\ L_{1\theta} & L_{2\theta} & L_{F\theta} & 0 & 0 \end{vmatrix} \qquad \text{(A-1)}$$

It will be sufficient to ensure that the solutions given in Section II-A describe a (the) minimum cost configuration if the determinants D_5 and D_4 are both positive, where D_4 is the 4 x 4 determinant that remains when the first row and first column of D_5 are removed. If one now computes the requisite partials of the L function and, without first evaluating them at the solution point, inserts them into their appropriate positions, D_5 becomes

$$D_5 = \begin{vmatrix} 0 & -\lambda a_1 & 0 & -a_1 x_2 & n \\ -\lambda a_1 & 0 & -\lambda a_2 & -Y & -1/m \\ 0 & -\lambda a_2 & 0 & -a_2 x_2 & 0 \\ -a_1 x_2 & -Y & -a_2 x_2 & 0 & 0 \\ n & -1/m & 0 & 0 & 0 \end{vmatrix} \qquad \text{(A-2)}$$

(where $Y = a_1 x_1 + a_2 F$), and D_4 is the "lower right 4 x 4 corner" portion of D_5 obtained if one crosses out its Row 1 and Column 1.

Omitting all tedious algebra details of the cofactor expansions of the two determinants, these are the results:

$D_5 = 2a_2^2 \lambda x_2 n(nY + 2a_1 x_2 /m) > 0$, and

$D_4 = (a_2 x_2 /m)^2 > 0.$

The fact that all components are positive assures $D_5 > 0$,[3/] while $D_4 > 0$ follows trivially since all its components are squared.

Thus, the second-order sufficient conditions are satisfied without having to "plug in" the actual solutions. The derived cost function is verified.

[3/] The non-negativity of λ is established by Equation (9), where $\lambda = r/a_2 x_2 > 0$ since $x_2 > 0$.

VOLUME THREE

WATER DEMAND FUNCTIONS: EMBEDDING QUALITY PARAMETERS AND EQUILIBRIUM ANALYSIS

VOLUME THREE

TABLE OF CONTENTS

I. INTRODUCTION

This volume presents the results of the second and final phase of a study for the U.S. Army Corps of Engineers Institute for Water Resources (IWR) under Contract No. DACA 71-72-C-0053. Volumes One and Two developed methodology (and numerical examples thereof) for explicitly incorporating quality parameters into water supply functions. In general terms, a classical economics model (CEM) showed how a cost-minimization analysis that is subject to a water quality side constraint can lead to deriving treatment plant total and marginal economic cost functions with a quality parameter embedded explicitly in each. Since the economic behavior of a plant would dictate using its marginal cost curve to help decide at what (e.g., daily) flow rate to operate profitably, such a function constitutes, in effect, the plant's supply function of treated water. By examining how the marginal cost function shifts in response to changes in the embedded quality parameter(s), one can determine how such changes influence a water supply function. This concept, along with numerical illustrations, is presented in the previous volumes.

Demonstrating a different approach to the question, a linear programming model (LPM) was formulated in order to show how a proven technique (solution algorithms are readily available) can be used to examine water supply within a resource allocation context. The LPM permits looking at large-scale problems and has as its central focus a representation of how a water-using producer would adjust his usage of high quality vs. low quality water in response to various parameter changes. In this way, the LPM measures how supplies of usable water are adjusted "internally" by a producer as different quality-related factors are altered.

OVERVIEW OF THE DEMAND SIDE

This volume focuses on the "other" side of the market, i.e., it is primarily concerned with developing quality concepts with respect to water demand functions. Thus, the main purpose is to show that it is possible to conceive of demand functions for water which explicitly incorporate quality parameters, as was the case for treatment supply functions. Once this is accomplished, the results then can be combined with the supply results from Volumes One and Two to arrive at a representation of water use equilibrium that is dependent on quality factors. This latter notion is also covered in this volume. It thus is possible to evaluate changes in equilibrium water amounts due to changes in quality. In so doing, the analysis is made more realistic because the term "water supply" is often understood to connote simultaneous action of demand as well as supply forces.

The methodology used here is again that of constrained optimization, to depict traditional economic theories of demand function derivation by utility maximization (for final consumers) and profit maximization (for water-using producers). For each of these demand forms, therefore, a conceptual model with explicit side constraint for allowable turbidity concentration is presented. This demonstrates the theory involved and, at the same time, serves as a general framework within which to develop all models that deal with the same question. Following this presentation, an example is given in each case to illustrate how the different demand curves are actually derived. These examples are hypothetical in that they use hypothetical utility and production functions, respectively, but they clearly indicate derivational procedures. In addition, all functions postulated and derived have intuitively realistic properties.

After deducing each demand curve, it is shown that a "comparative statics" analysis enables evaluating how the demand curve would shift if the allowed turbidity concentration changed. And, finally, a similar procedure measures what the effect of changing this quality parameter has on the "quantity demanded = quantity supplied" equilibrium that results when Volume Two's derived CEM marginal cost function is used in connection with any demand function.

There is no econometric or empirical analysis in this volume, although means of implementing the results derived are clearly delineated. For this reason, Volume Three is like Volume One but differs from Volume Two (in which some statistical results were reported); the current volume being a documentation of conceptual methodology accompanied by postulated examples to make the analysis more concrete and lucid.

CONTENTS OF VOLUME THREE

References have already been made to the contents of subsequent parts of this last volume. Specifically, Chapter II develops the economic theory of consumer demand to show how a quality parameter can effect changes in the demand for water by "final use" consumers. An analogous discussion for water-using producers (input, or "derived," demand, in economics terminology) follows in Chapter III, while Chapter IV combines the results of the two previous chapters to arrive at a representation of aggregate demand. In each chapter, a hypothetical example demand function is derived and then analyzed in connection with Volume Two's CEM marginal cost function to demonstrate the concept of equilibrium water supply.

II. CONSUMER WATER DEMAND FUNCTIONS

Two possible reasons can be distinguished for wanting (having demand for) something: it is desired as an end by itself (final consumption), or it is intended for intermediate use in helping derive other items (input consumption). Recognizing that there are fundamental differences between the motivating factors in each instance, traditional economic theory addresses each by means of a separate analysis. Accordingly, two types of demand for water will be addressed: this chapter covers the first type, termed consumer demand; Chapter III develops demand concepts for water used as an input.

A. UTILITY-BASED DERIVATION WITH A QUALITY CONSTRAINT

The usual simplified but intuitively reasonable representation of consumer behavior in acquiring goods is that a person purchases a mix of items per unit time so as to maximize the satisfaction (utility) he realizes without exceeding the budget he has available to spend.[1/] From such a verbal description, one can readily deduce an appropriate analytical model depicting utility maximization subject to a budget constraint which incorporates

1/ It is neither the purpose nor intent of this study to survey alternative theories of consumer behavior. For information in that regard, as well as discussion of the traditional theory used here, see any recognized source such as Horowitz (6), Henderson and Quandt (5), Ferguson (4), or Baumol (2), where parenthesized numbers are bibliographical entries for this volume.

prices of the items being consumed and spendable income. When this constrained maximization model is solved, the resulting optimal amounts of the goods are functions of all parameters in the model, including each good's own price. As such, these solutions are demand functions because they show what quantity of each good is desired at any given price (of it and related goods), and income level.

If two of the "goods" being consumed are water and some contaminant(s) present in the water, and, in addition, if a side constraint to reflect desired water quality is imposed, then an analysis directly analogous to the cost function derivations of CEM in Volumes One and Two results. That is, utility maximization subject to budget and quality constraints is advocated here, as contrasted with CEM's cost minimization subject to output and quality constraints. However, the methodology for both models is the same.[2]

Turning to a more explicit rendering of these points, define the following variables and parameters for a particular individual, "Mr. i":

G = Quantity of an all-purpose product ("Good") consumed by Mr. i per time period

q; T = Quantities of water and contaminant, respectively, consumed by Mr. i per time period

p_g, p_w = Unit prices of Good and water, respectively

M = Amount of spendable income Mr. i has

2/ See the sources listed in Footnote 1 for more discussion about the existence of utility functions, including the distinction between cardinality and ordinality. It should be noted that, in the present study, the utility function is merely a "means to an end" and not an end in itself; hence, its empirically demonstrable existence is not critical. That is, so long as as the notion is conceptually well-defined, then it is sufficient to justify "end result" demand curve properties that can be deduced. It is the demand curve that can then be tested empirically.

$U(G, q, T)$ = Mr. i's utility, or preference-ordering, function. $U(\ )$ is assumed to be a twice-differentiable, ordinal ranking function with positive first partials ("positive marginal utilities") but negative second partials for q and G, and just the opposite for T.[3/]

$f(q,T)$ = A monitoring quality indicator function strictly comparable to that in Volumes One and Two. It represents a means of measuring quality in terms of T as related to q as, for example, the concentration ratio T/q that will be used in the illustrations throughout this volume, although $f(\ )$ need not necessarily be of such form.

With these definitions, Mr. i wants to consume quantities q, G, and T so as to--

$$\text{maximize utility} = U(q, G, T) \tag{1}$$

subject to:

$$M = p_g G + p_w q \tag{2}$$

and

$$f(q,T) = s \tag{3}$$

Equation (2) is Mr. i's budget constraint, depicting that he exhausts his spendable income on his purchases of Good and water (he does not "purchase" the the contaminant T *per se*; it comes "free" with q). Equation (3) stipulates that the quality of the water he uses (where quality is measured by the $f(\ )$ function) must equal the value s, an externally specified standard (e.g., a fecal coliform count drinking water standard).

As shown in Volumes One and Two, optimization by the method of Lagrange multipliers is used to solve the posed problem in an efficient

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3/ Using subscript notation for partial derivatives, the first partial $U_G > 0$ indicates that Good is a desirable product (more of it is preferred to less), but the second partial $U_{GG} < 0$ denotes that as additional amounts of it are consumed, the increments in satisfaction become smaller ("diminishing marginal utility"). On the other hand, $U_T < 0$ and $U_{TT} > 0$ say that the contaminant is an undesirable entity (utility declines as T rises) and becomes more so as more of it is consumed.

and mathematically correct way.[4/] Thus, the following Lagrangian function is formulated, where λ and μ are to-be-determined (endogenous) multipliers:

$$L = U(q, G, T) + \lambda[M - p_g G - p_w q] + \mu[f(q, T) - s] \qquad (4)$$

The function L is maximized with respect to the three decision variables (q, G, T) and two multipliers by finding what values of them will cause their five respective partial derivatives of L to equal zero simultaneously. That is, the "first order" optimization conditions are a system of five simultaneous equations of form $L_k = 0$, where the L_k's are the partials of L with respect to the five endogenous variables. Letting subscripts on U and f denote partials, the system appears as:

$$U_G - \lambda p_g = 0 \qquad (5)$$

$$U_q - \lambda p_w + \mu f_q = 0 \qquad (6)$$

$$U_T + \mu f_T = 0 \qquad (7)$$

Equation (2) (8)

Equation (3) (9)

These five equations are conceptually solvable for the five endogenous variables as functions of the prices p_g and p_w; income M; and the quality parameter s. Of special relevance here would be the solution for the optimal desired quantity of water which is written schematically as

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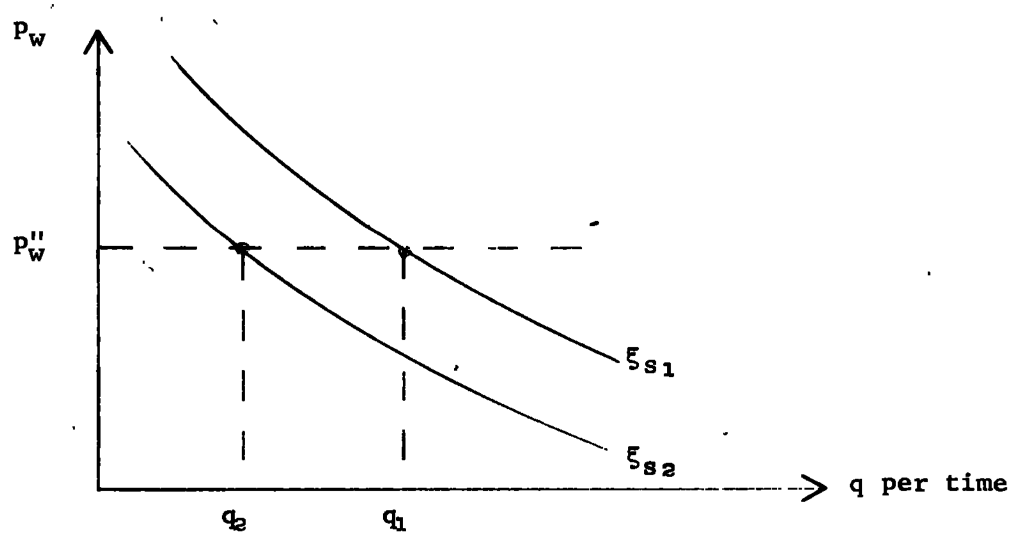
4/ When constraints are simple enough in form to permit solving for one variable in terms of others, then such "solution" can be substituted directly into the function to be optimized with the result that (1) the constraint has been explicitly accounted for, and (2) one endogenous variable has been eliminated from the optimization process. If this procedure can be used to eliminate all but one variable in an objective function, then the Lagrangian process is unnecessary. Since such complete "reduction" is usually not the case, and, furthermore, because the Lagrangian process is a general technique, it is presented here.

$$q = \xi(p_w;\ p_g,\ M,\ s,\ \text{other parameters}) \tag{10}$$

and indeed constitutes Mr. i's demand function for water. With s appearing explicitly, it is clear that the effect on q due to a change in quality can be ascertained by evaluating the partial derivative of ξ with respect to s, namely $\frac{\partial \xi}{\partial s}$. In more customary economics terms, this measures how Mr. i's demand curve for water shifts in response to a specified change in quality when all other factors are held fixed. The familiar "price vs. quantity" diagrams of Figure II-1 illustrate the hypothetical case of an outward shift in demand ("increased demand") with more stringent quality standards (s_1 reflects a more stringent standard than does s_2). It shows that at any postulated price, say p_w'', Mr. i would want to consume q_2 amount of s_2 - quality water,

FIGURE II-1

INCREASED DEMAND FOR WATER BY MR. i DUE TO STRICTER QUALITY STANDARDS



but if the quality standard increased to s_1, then Mr. i would want to use q_1 amount. The difference (q_1 - q_2) represents $\frac{\partial \xi}{\partial s}$ if (s_1 - s_2) is a "unit" change in quality standard. Observe that both curves are drawn downward-sloping to depict the intuited characteristic of demand curves that greater quantities are demanded at low prices than at high prices.

Succinctly summarized, Equation (10) (along with the model underlying it) shows that it is very plausible to visualize consumer demand functions for water that account explicitly for quality parameters. So long as a consumer can be expected to pay attention to quality aspects in his behavior patterns, then a constrained optimization model will indeed give rise to a demand relation which accounts for this fact.

B. HOW DOES DEMAND SHIFT?--THE METHOD OF COMPARATIVE STATICS

Although the previous section sets forth the conceptual basis for associating demand with quality, it leaves open the question of whether it is possible to tell a priori how the demand curve will shift when there is a change in the quality standard(s). In terms of Figure II-1, is there any way to predict ahead of time if the $\xi(\ )$ function will move inward or outward as the parameter s changes? One can answer in the affirmative that a methodology for evaluating such shifts does exist, and it does not require knowing the demand function itself (a fact that can be significant for empirical operationality). In other words, $\frac{\partial q}{\partial s}$ can be computed without first knowing Equation (10).

Known to economists as comparative statics analysis,[5] this method for deducing properties of a function without knowing the function itself involves computing derivatives directly from the underlying ("static," or "equilibrium") system which gives rise to the ultimately desired solution functions. This can be done primarily due to the linearity property of

[5] See Chiang (3), Chapters 6-8 and 11, for an introduction to comparative statics analysis.

differentials and is best demonstrated by working with the model in the previous section but in a more specific form.

Refer, therefore, back to Equations (1)-(3), but let Equation (3)'s quality constraint be rendered as an allowed concentration of turbidity contaminant per unit of water consumed, analogous to the CEM illustrative supply function example of Volume Two (this similarity is purposeful, for reasons of consistency in integrating the supply and demand analyses, as will be seen). Thus,

$$T/q = s \quad ; \quad \text{or} \quad T = sq \tag{3'}$$

In other words, (3') says that Mr. i consumes utility maximizing amounts of water and turbidity, but only if the ratio of the latter to the former equals a pre-determined standard s.

Next, "solve" Equation (2) for G as a function of q:

$$G = (M - p_w q)/p_g \quad . \tag{2'}$$

With both T and G expressed now as functions of q , the utility function of Equation (1) can be written solely in terms of the decision variable q , and both budget and quality constraints will have been accounted for:

$$U\,[(M - p_w q)/p_g;\ q;\ sq] \quad . \tag{1'}$$

Since only one variable remains in the objective function and both constraints have been incorporated explicitly, the single-variable optimization alluded to in Footnote 4 can be implemented. Analogous to Equations (5)-(9), the relevant first-order condition here is to set the first derivative of (1') with respect to q equal to 0:[6/]

6/ Calculating $\frac{dU}{dq}$ involves computing the total differential of Equation (1) as

$$dU = U_q d_q + U_G dG + U_T dT \quad ,$$

dividing through by dq to form derivatives on both sides of the equation, and then inserting $\frac{dG}{dq}$ and $\frac{dT}{dq}$ as computed from (2') and (3'), respectively.

$$-(p_w/p_g) \cdot U_G + U_q + sU_T = 0 \tag{11}$$

where U_G, U_q, and U_T are again the partial derivatives of U with respect to G, q, and T.

Equation (11) is Mr. i's "equilibrium" equation because, when solved, it will yield his utility-maximizing value of q, and, according to the behavior assumption made, when he has maximized his utility, he has indeed achieved his optimum (equilibrium) consumption mix. In other words, even without solving for q, it is known that Equation (11) represents the optimum, for this is the equation that must be satisfied by the optimum q value. It is for this reason, therefore, that comparative statics analysis asserts that changes in optimum q can be deduced by differentiating the first-order condition, namely Equation (11), to isolate the derivative of q with respect to any desired parameter. Applying this reasoning, the effect of a change in allowed turbidity concentration s on Mr. i's level of water demanded is given by[7/]

$$\frac{\partial q}{\partial s} = \frac{-q[-(p_w/p_g)\, U_{GT} + U_{qT} + sU_{TT}]}{D^2 U} \tag{12}$$

where the U_{ij} terms are second-order partial derivatives of U, and D^2U is the second total derivative of U with respect to q. All the derivatives are evaluated with the optimal q value.

Equation (12) thus shows that if the signs/values of various second partials are known a priori, then one can know the direction/magnitude of a "representative individual's" water demand shift in response to increased or decreased allowed turbidity concentration without first needing to derive the relevant demand curve. If, for instance, the right side of (12) is known

7/ See Appendix A for the essential details leading to this derivation.

to be positive, then one can predict that a more stringent turbidity standard (s falls) would cause a decline in water quantity demanded, other things being the same. In fact, however, the sign of (12) is ambiguous; meaning that q might either rise or fall as s falls. More specifically, the denominator D^2 can be expected to be negative if one assumes the second-order (sufficient, not necessary) conditions for maximizing a function of one variable are met, but the numerator has an ambiguous sign. Footnote 3 indicated $U_{TT} > 0$ is most plausible, but nothing specific can be stipulated about U_{GT} and U_{qT}. Only if (1) the former is negative while the latter is positive, or (2) both are zero, can one predict a priori (without computing relative magnitudes first) the sign of $\frac{\partial q}{\partial s}$, which in these cases would be positive. The intuitive interpretation of (1) is that the rates at which water and Good 1 enhance Mr. i's satisfaction (i.e., their "marginal utilities") increase and decline, respectively, as more turbidity is consumed, while (2) would mean that greater turbidity consumption has no effect at all on the marginal utilities of water and Good. Since one has no reason to assert categorically the veracity of either of these cases, it must be concluded that comparative statics analysis yields an ambiguous sign on $\frac{\partial q}{\partial s}$. In other words, economic theory does not tell us "ahead of time" to expect one result or another; specific situations must be examined individually. As a means of doing this, however, the comparative statics technique is indeed an appropriate methodology to use when the actual demand function is not known.

C. CONSUMER DEMAND--HYPOTHETICAL EXAMPLE

If one postulates a hypothetical utility function to play the role of Equation (1), then the essential features of the previous sections can be clearly demonstrated by hypothetical example. In this section, therefore, the constrained utility-maximization procedure will be used to derive a demand function in which a quality parameter (again to be rendered as turbidity concentration) appears explicitly. It will then be possible to determine how the demand curve shifts in response to changes in the quality parameter, and finally, a notion of equilibrium water as a function of quality will be illustrated by combining this analysis with Volume Two's derived marginal cost function.

Demand and Marginal Revenue Functions

Suppose it is known that the following utility function characterizes Mr. i's preference ordering process:

$$U = [(q^2/T) + \ln(G)]^{\alpha} \quad , \text{ where } \alpha < 1 \quad . \tag{13}$$

By computing appropriate partial derivatives, this function can be shown to exhibit the intuited properties discussed in Footnote 3 of Section II-A. Hence, water and Good are desired consumable items (marginal utilities U_q, $U_G > 0$), but turbidity is not "liked" ($U_T < 0$); the first two are characterized by diminishing marginal utilities U_{GG}, $U_{qq} < 0$, while turbidity has $U_{TT} > 0$. In addition, the fact that the various cross-partials of form U_{ij} are non-zero means that the three consumed items are not independent of each other, i.e., Mr. i cannot augment his satisfaction by consuming more and more of any one of the three to the total exclusion of either of the other two.[8/]

Having thus established that, although strictly hypothetical, Equation (13) satisfies restrictions that make it intuitively plausible, one

8/ This property is often referred to as the "non-additivity" of utility--see Ferguson (4), pp. 21-22.

can proceed to set up the constraints against which maximization (13) takes place. In fact, however, this has already been done, for Equations (2') and (3') of the previous section gave the explicit forms of, respectively, the budget and quality (turbidity concentration) constraints to be used. Re-stated together, the model being analyzed is to maximize (with respect to q, G, and T)

$$U = [(q^2/T) + \ln(G)]^{\alpha} \qquad (\alpha < 1) \tag{13}$$

subject to:

$$G = (M - p_w q)/p_g \qquad \text{(budget constraint solved for } G\text{)} \tag{2'}$$

and

$$T = sq \qquad \text{(turbidity constraint solved for } T\text{).} \tag{3'}$$

As was discussed and shown before, when the constraints in a model are readily solvable in terms of other decision variables, and there are enough constraints to enable eliminating all but one variable in the objective function, then single-variable unconstrained optimization can be applied instead of the Lagrange multiplier technique. Substituting (2') and (3') into (13) thus gives U as a function of only q:

$$U = [q/s + \ln\{(M - p_w q)/p_g\}]^{\alpha} \quad . \tag{13}$$

Computing the derivative of (13') with respect to q, setting it equal to zero, and solving the resulting equation for q gives a particularly simple expression for Mr. i's water demand function (see Appendix A for derivation details):

$$q = (M/p_w) - s \quad . \tag{14}$$

Intuitively sensible, Equation (14) shows that q is directly related to income M, but inversely related to water price p_w (the latter signifying the usually-presumed downward-sloping property of demand curves).

Thus, Equation (14) shows what quantity of water Mr. i will want to consume, given specified price, income and turbidity concentration values, and assuming the utility function of Equation (13).

Referring again to Volume One, it will be recalled that the theory developed there of how a treatment plant decides what quantity of water to "produce" showed that a rational profit-maximizing decision must rely on the marginal revenue concept. By definition, marginal revenue (MR) is the derivative of total revenue (TR) with respect to quantity, where $TR = p_w \cdot q$; and p_w is the unit price of water a consumer is willing to pay; in other words, price, as given by the demand curve. For the case at hand, the latter is easily found by solving Equation (14) for p_w as a function of q (sometimes referred to as the "inverse demand function"):

$$p_w = M/(q + s) \qquad (15)$$

It follows then that

$$TR = p_w \cdot q = Mq/(q + s) \qquad (16)$$

gives total revenue as a function of q, and

$$MR = \frac{\partial TR}{\partial q} = sM/(q + s)^2 \qquad (17)$$

is the marginal revenue function corresponding to Mr. i's demand function. Equation (17) thus shows what would be the incremental revenue realized when an additional unit of water is consumed by Mr. i. Observe that MR is always positive and declines monitonically with q for this example--properties which are consistent with economic theory.

Shifts in Demand Due to Quality Changes

Because the demand function has been derived explicitly, it is not necessary, as noted before, to use comparative statics analysis for evaluating the effect of a change in turbidity concentration on demand. Rather, the partial derivative of q with respect to s can be computed directly, giving simply

$$\frac{\partial q}{\partial s} = -1 < 0 \quad , \tag{18}$$

which shows that in this example, a more stringent quality constraint (s falls) leads to increased demand (recall Figure II-1). Similar computation for the marginal revenue function, on the other hand, reveals an ambiguous sign:

$$\frac{\partial MR}{\partial s} = \frac{M(q - s)}{(q + s)^3} \quad \text{, which} \tag{19}$$

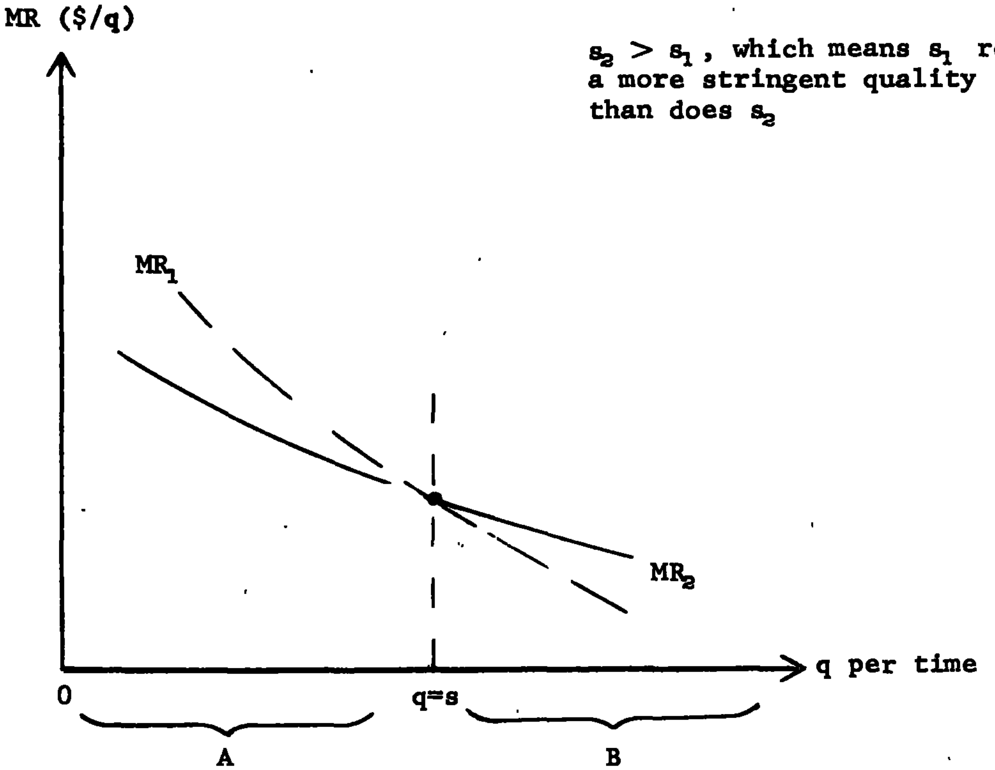
can be positive or negative, depending on the value of q relative to s , namely if $q > s$ or $q < s$, respectively. In general, therefore, for large q values, the tendency would be for the example MR function to shift inward with a fall in s , while the opposite would occur if q is small. Graphically, such property is depicted by "pivoting" the MR curve so that one curve representing a lower value for s intersects a curve representing a higher s value from above, where q equals s. Figure II-2 illustrates this representation which, it should be re-emphasized, is characteristic of the specific hypothetical demand function being analyzed and not meant to reflect properties of "any" consumer water demand function.

It is thus seen that, just as it was possible from Volume Two to compute shifts in total and marginal cost curves as a means of evaluating the effect of quality parameter changes on water supply functions, so too can quality-related demand and marginal revenue changes be computed. The degree to which such shifts can be measured empirically depends, of course, on how well-specified the demand and marginal revenue counterparts of Equations (14) and (17) are. There is no a priori reason to deduce from the simple forms presented here that they would be typical of what one would derive in an actual application. It should be recalled, however, that comparative statics

FIGURE II-2

SKETCH OF MARGINAL REVENUE CURVE FOR HYPOTHETICAL ILLUSTRATIVE CONSUMER DEMAND FUNCTION*/

$$MR = sM/(q + s)^2$$



Since $q < s$ in Region A, $\frac{\partial MR}{\partial s} < 0$ by Equation (19). Hence, a decline from s_2 to s_1 (imposition of more stringent quality standard) causes an upward shift in MR from MR_2 to MR_1. In Region B, the shift is downward because $q > s$.

*/ As is usually the case, one cannot depict "instantaneous change" calculus results exactly in diagrammatic terms. Thus, the pivoting characteristic shown represents, but does not measure exactly, the $\frac{\partial MR}{\partial s}$ value from Equation (19) because, strictly speaking, the diagram is really valid for only a small range of q and s values. This point is further illustrated by the fact that the derivative formula does not specify which of s_1 or s_2 to use for s in Equation (19). Hence, one cannot depict unambiguously where $q = s$.

analysis can be used when the demand function is not readily determinate. On the other hand, it was seen that the comparative statics approach requires knowledge of a preference ordering function which is at best a conceptual notion that is not practical (even if possible) to quantify. In short, analytic derivation of a consumer demand function, while a theoretically very well-defined process, may be virtually impossible to implement.

For this reason, it is most likely that one would think in terms of estimation procedures to deduce a demand function directly (Equation (10), that is), rather than by explicit derivation from the constrained utility maximization model.[9/] Regression methods, for example, might be used analogous to the suggested "direct" estimation of cost functions in Volume Two. Methodology for doing this is well-documented in econometrics literature;[10/] the major potential problem would be obtaining the personal data observations requisite for "Mr. i's" own individual demand function. If (as is most plausible to expect) one thinks in terms of aggregate demand, however, then this latter issue becomes essentially moot, assuming aggregate data for estimation are available. The structure supporting such an examination is developed in the following subsection.

Equilibrium Aggregate Water Quantity and the Effect of Quality Changes

In an attempt to make the analysis developed thus far more realistic, assume that there are k' total consumers, each with a utility function

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9/ Some care should be exercised in choosing a function form for estimation to be sure that it would not imply inconsistency with intuited properties of any implicit underlying utility function. This whole question of deducing the underlying utility function from a known demand function is known to economic theorists as the "integrability problem," information about which can be found in R. D. G. Allen (1).

10/ See, for example, Johnston (7), Chapter 12.

similar in form to Mr. i's of Equation (13), but not necessarily identical to it (for example, the α exponent may differ from person to person). In addition, assume each consumer has his own spendable income (M_i) and tolerance level turbidity constraint s_i. With these assumptions, a model for each consumer can be formed as Equations (13), (2') and (3') where all symbols except market prices p_w and p_g would have "i" subscripts attached. The water demand solution for "each Mr. i" is thus Equation (14), with subscripts:

$$q_i = (M_i / p_w) - s_i \quad .$$

Aggregate water quantity demanded by consumers at any price is now simply the sum of the q_i's:

$$q_C = \Sigma q_i = (\Sigma M_i)/p_w - \Sigma s_i$$

where the sums go up through $i = k'$. Letting $M' = \Sigma M_i$ and $s' = (\Sigma s_i)/k'$ for notational efficiency (thus, M' is total aggregate spendable income, while s' is the average turbidity standard for all the consumers), then the aggregate illustrative example demand function can be written as:

$$q_C = (M'/p_w) - s'k' \quad . \tag{20}$$

The aggregate demand function therefore has a form virtually the same as the individual demand function. Accordingly, it would be expected that the marginal revenue function corresponding to Equation (20) would be similar in form to Equation (17), and it is:

$$CMR = (s'k'M')/(q_c + s'k')^2 \quad . \tag{21}$$

<u>Figure II-2</u> can easily represent Equation (21) if the symbols q, s, and MR are changed to be q_c, $s'k'$, and CMR, respectively. In other words, as s' falls, CMR rises if $q_c < s'k'$, but falls when the inequality is reversed.

With a representation of aggregate consumer demand thus established, it is now possible to examine ways in which consumer demand forces may operate in conjunction with Volume One's supply concepts to give an indication of how quality changes can affect "equlibrium" water quantity. Recalling the previous discussion about the characterization of a treatment plant's (profit-maximizing) optimization, or equilibrium, behavior, then one can depict the familiar "marginal revenue = marginal cost" result of economic theory in schematic terms as:

$$CMR(q, s', V_D) = MC(q, s', V_S) \tag{22}$$

where these are meant to be general representations of consumer marginal revenue as a function of quantity per time q, turbidity standard s, and other demand-related variables V_D ; and marginal cost as a function of q, s', and other supply-related variables V_S. Common symbols for q and s' have purposely been used in both functions to show that the two have the same dimensions and are indeed functions of the same variables; it therefore makes sense to equate them. In other words, at equilibrium, water quantity "demanded" by consumers (q_c) is identical to quantity "supplied" by a treatment plant (q^s, say), so a common symbol can be used. Similarly, referring to Volume Two's CEM example, where post-treatment turbidity concentration was taken as the quality parameter in the cost function derivation, the value denoted by s' is appropriate for both functions because it measures the same thing in each instance.

If specific functions are given for CMR( ) and MC( ) , then it may be possible to solve the equilibrium Equation (22) for an explicit q value. The effect of a change in quality standard s' on equilibrium water quantity can then be found by computing $\frac{\partial q}{\partial s'}$.

On the other hand, when functions are specified but an exact solution for q cannot be obtained, then a comparative statics analysis like that of Section II-B can be performed to evaluate $\frac{\partial q}{\partial s'}$ (such is the case for the example presented below). The analysis proceeds by computing the total differential of the equilibrium equation, collecting terms in dq, forming derivatives with respect to s', and then solving for $\frac{\partial q}{\partial s'}$ by holding fixed all variables other than q. The result will be a solution expressed in terms of partial derivative characteristics of the marginal revenue and cost functions. As an illustration, the comparative statics solution for the general formulation of Equation (22) is given by

$$\frac{\partial q}{\partial s'} = \frac{C_2 - R_2}{R_1 - C_1} \tag{23}$$

where R_i and C_i are partial derivatives of CMR( ) and MC( ), respectively, with respect to q (i=1) and s' (i=2).[11/]

Equation (23) indicates that a change in s' has an ambiguous effect on equilibrium q when it is realized that all the analyses developed to this point suggest that the effect on marginal revenue of a change in s' is itself of uncertain sign. That is, R_2 may be positive, negative or zero; this alone is sufficient to render Equation (23) of ambiguous sign. In addition, however, when both the marginal revenue and marginal cost (see Volume Two) are downward sloping with respect to q, R_1 and C_1 are then both negative, so the denominator of Equation (23) has an indeterminate sign. The case demonstrating the uncertain sign for $\frac{\partial q}{\partial s'}$ therefore becomes even stronger. Succinctly put, the methodology for evaluating the directional

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11/ The total differential of Equation (22) is

$$R_1\,dq + R_2\,ds' + R_3\,dV_D = C_1\,dq + C_2\,ds' + C_4\,dV_S$$

where the R_i and C_i terms are defined in the text, except that R_3 and C_4 are partials with respect to V_D and V_S, respectively. From this equation, Equation (23) is readily deduced by holding V_D and V_S fixed.

effect of s' on q is well-defined, but one cannot assert what this effect will be on a priori theoretical grounds.

Returning now to the illustrative example, these results can be demonstrated with actual functions. Assuming, therefore, that a single central treatment facility, whose marginal cost function is Equation (16) of Volume Two,[12] serves a population of k' water-consuming persons characterized by the aggregate demand curve of Equation (20) above, then the equilibrium equation counterpart of Equation (22) is:

$$s'A + D/2\sqrt{q} = (s'k'M')/(q + s'k')^2 \quad . \qquad (24)$$

Equation (24) can be transformed into a fifth degree polynomial which, if data were available to quantify it, could be solved by numerical methods to obtain a numerical value for q. However, it is not possible to solve Equation (24) analytically to obtain q as a mathematical function of s' (and other parameters). Consequently, the comparative statics approach alluded to above must be used to calculate $\frac{\partial q}{\partial s'}$. Without presenting the algebraic details of the derivation, the final result is:

$$\frac{\partial q}{\partial s'} = \frac{\eta_1 - k'\eta_2}{\eta_2 - \eta_1 s'/2} \qquad (25)$$

where $\eta_1 = D/2\sqrt{q}\ (s')^2$ and $\eta_2 = 2k'M'/(q \pm s'k')^3$. Consistent with the theory result relevant to Equation (23), the sign of Equation (25) is ambiguous since the η_i terms are each positive but their relative magnitudes are not known a priori. Actual numerical values of the parameters and q would have to be inserted in order to quantify Equation (25), and such evaluation was not part of this study. (Volume Two, of course, did quantify an example of the marginal cost function, but no empirical analysis was specified for marginal revenue.) It is nonetheless possible to indicate graphically some of the results that could occur with these example functions.

12/ Volume Two's Equation (16) is: $MC = sA + D/2\sqrt{q}$, where $A < 0$ and $B > 0$ are parameters. Following the text discussion, the turbidity concentration parameter s becomes s' in the equilibrium analysis here.

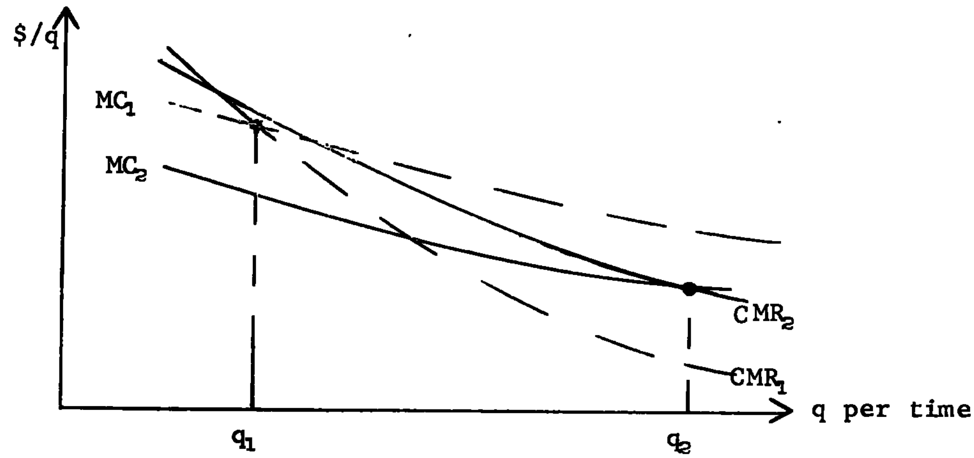
Using Volume Two's Figure II-2 as a basis for sketching marginal cost curves arising from the function used here and this volume's Figure II-2 as a model for general shapes of the example marginal revenue function, two cases can be drawn to illustrate possible situations. These appear in Figure II-3, where it is assumed that a more stringent turbidity standard ($s' \downarrow$) causes increased marginal cost ($\frac{\partial MC}{\partial s} < 0$) as was the case with the numerical example in Volume Two. In Figure II-3a, the case shown depicts equilibrium occurring in the region where marginal revenue falls with a drop in s'. It is seen here that equilibrium water quantity unambiguously falls, from q_2 to q_1. Figure II-3b, on the other hand, depicts equilibria where marginal revenue rises in response to a fall in s'. In this case, the ultimate effect on equilibrium water quantity is ambiguous, depending on relative magnitudes of the function shifts. The diagram indicates that q may rise, fall, or not change at all.

Thus, it is seen that consumer water demand functions which account explicitly for quality factors are indeed conceptually well-defined, as is the methodology for evaluating how a change in quality factor(s) will affect consumer demand. In addition, the notion of equilibrium consumer water quantity makes sense, and it too can be evaluated in response to quality change. The purpose of the following chapter is to show that similar results can be deduced for water-using producers.

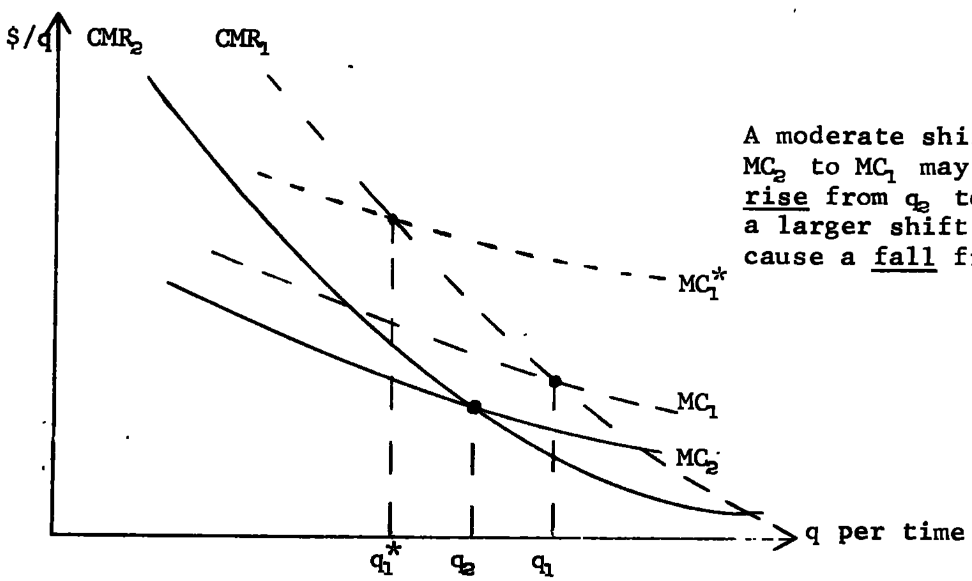
FIGURE II-3

SKETCHES OF TWO POSSIBLE CASES OF QUALITY EFFECT ON EQUILIBRIUM WATER IN HYPOTHETICAL ILLUSTRATIVE EXAMPLE
(More Stringent Turbidity Standard is Depicted by Shifting From Subscripts 2 to Subscripts 1)

II-3a. Equilibria where $\frac{\partial CMR}{\partial s'} > 0$: unambiguous fall from q_2 to q_1 as s' falls.



II-3b. Equilibria where $\frac{\partial CMR}{\partial s'} < 0$: ambiguous change in q as s' falls.



A moderate shift from MC_2 to MC_1 may cause a rise from q_2 to q_1, but a larger shift to MC_1^* may cause a fall from q_2 to q_1^*

III. PRODUCER DERIVED DEMAND FUNCTIONS FOR WATER

It is traditional in economic theory to recognize that a producer who transforms inputs into output "derives" his demand for the inputs he uses from consideration of (1) his production process, and (2) the market demand for his product. It is intuitively reasonable that this would be the case, for (1) determines the physical efficiency with which the inputs are used, while (2), in reflecting the rate at which the produced item can be marketed, helps determine the rate at which production should occur. Both factors are therefore inherently important to the input use decision and characterize what is meant by "derived demand."

A. QUALITY-CONSTRAINED PROFIT-MAXIMIZATION AND CONCEPTUAL DERIVED DEMAND SHIFTS

An appropriate model for capturing the points just discussed is to depict a producer as a profit-maximizing entity who purchases his inputs at known prices (or supply functions), sells his output at a specified price (or according to a known demand function), and uses a prescribed production process.[13/] Applying this description to the question at hand, we can construct a model with a single-impairment (which will again be turbidity) to examine the essential topics. Therefore, let

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13/ Horowitz (6), Chapters 10 and 11, discusses new theories which are variations on, or alternative to, the neoclassical profit-maximization, but the latter is very adequate for presenting the derived demand concept.

$z \equiv Z(q, Y, T)$ = a specified producer's output per time period of Product z, as generated by production function $Z(\;)$ from flows of water (q) and other (Y) inputs. It will be assumed that q and Y have positive ($Z_i > 0$) but declining marginal products ($Z_{ii} < 0$; $i = q, Y$), while the opposite holds for turbidity amount T (the subscript notations denote partial derivatives).

T = amount of turbidity flowing per time period into the production process

p_z; p_w; p_y = unit prices of Product z, water, and other inputs, respectively. These prices may be parametric or may represent demand (p_z as a function of z) or supply (p_w and p_y as functions of q and Y) functions, depending on prescribed specification.

s = the producer's turbidity concentration tolerance, given in terms of turbidity amount per unit amount of water (q) used;14/ thus, the producer is restricted to allowing turbidity inflow only in the amount $(T/q) = s$.

The producer's static profit objective function is simply revenue net of input expenditures; symbolically:

$$\pi = p_z \cdot Z(q, Y, T) - p_y Y - p_w q \qquad (26)$$

Profit-maximization behavior would therefore dictate maximizing (26) with respect to the flows of q, Y and T , but subject to the turbidity constraint outlined above, namely

$$(T/q) = s, \text{ or } T = sq \qquad (27)$$

As was the case in Chapter II's consumer demand analysis, direct substitution of (27) into (26) above is possible and obviates the need to use Lagrange multipliers, although, unlike the previous analysis, the model here cannot be reduced to a single-variable optimization. That is, using (27) renders (26)

14/ Other forms of quality constraint could be constructed, including a general conceptual formulation analogous to Equation (3). Efficiency is gained, however, by introducing immediately the form that will be used, because of its consistency so far with all the derivations as an example subsequently. There is no loss of generality in doing this for the methodology used is easily generalized.

as a function of q and Y alone, with the turbidity constraint explicitly incorporated; hence unconstrained maximization of

$$\pi = p_z \cdot Z(q, Y, sq) - p_y Y - p_w q \tag{26'}$$

can occur, but it is with respect to q and Y simultaneously.

Let p_z be the inverse demand function for Product z so that it is a function of Z, [15] but assume for now that p_y and p_w are parametric. With these stipulations, the first-order maximization equations resulting from setting partials of π (denoted π_i) with respect to q and Y equal to zero are:

$$\pi_Y = 0 \qquad MR_z \cdot Z_Y - p_y = 0 \tag{28}$$

$$\pi_q = 0 \qquad MR_z \cdot (Z_q + sZ_T) - p_w = 0 \tag{29}$$

where MR_z and Z_i are, respectively, the marginal revenue and marginal product expressions defined in Footnote 15 and, consequently, are themselves functions of only the variables q and y. This latter fact means that Equations (28) and (29) are a system of two simultaneous equations conceptually solvable for q and y as functions of the model's parameters; the solutions are precisely

[15] That is, $p_z = p_z(Z)$. Partial derivatives of p_z with respect to each production variable are thus calculable. Of particular relevance here, total revenue on sales of z is $p_z(Z) \cdot z$, and the partial derivative of revenue with respect to Variable i ($i = q, Y, T$), known as its "marginal revenue product," is written as

$$MRP_i = MR_z \cdot Z_i ,$$

where MR_z is the marginal revenue of Product z, and Z_i is the marginal product of Variable i. This expression is derived by chain rule differentiation of $p_z \cdot z$, one component of that operation being $MR_z = p_z + Zp_z'$, where p_z' is the total derivative of p_z with respect to z. Note that in (26') q appears in two positions in the Z function, meaning that chain rule differentiation has to be used to get $(Z_q + sZ_T)$ as the partial of Z with respect to q.

the producer's derived demand functions for these inputs.[16/] In schematic terms, the demand for water would be

$$q = \psi(p_w;\ p_y,\ s,\ \text{other parameters}) \tag{30}$$

where the presence of the turbidity concentration parameter s signals an explicit effect by it on q. As in the other cases examined in this study, therefore, the constrained optimization framework demonstrates the conceptual feasibility of a producer's demand function for water that explicitly accounts for a quality factor as well as a methodology for deriving such.

As was done in Chapter II, one can now examine the possibility that there may be *a priori* theoretical reasons for predicting that a change in s will cause $\psi(\ )$ to shift up or down, representing increased or decreased water demand in response to different quality factor values. Again, comparative statics analysis is employed, involving total differentials of Equations (28) and (29) and then the formation of appropriate derivatives

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16/ A technical point should be noted. The solutions here are indeed derived demand functions, for they describe the loci of optimal q and Y values, respectively, for *any given price levels* p_w and p_y. If these prices were themselves functions of the inputs, however, then the maximization procedure would give optimal input values, but each solution would not necessarily be an explicit function of its own price (since prices would not be parameters) and therefore would not be "quantity vs. price" derived demand function. It is thus correct to *assume* parametric input prices when deducing demand functions, but whether or not such solutions are the final profit-maximizing input values depends on whether the input prices *are actually* parametric. Strictly speaking, this same qualification could have been made for the utility-maximization consumer demand derivations, but there the case of non-parametric prices is less likely to arise since a consumer is less likely to be able to influence a price by adjusting the amount he buys than is a producer.

from those.[17] Using the efficient subscript notations of Footnote 17, the effect of s on q is measured by

$$\frac{\partial q}{\partial s} = \frac{\pi_{YY}\,\pi_{qs} - \pi_{Yq}\,\pi_{Ys}}{D_2} \tag{30}$$

where $D_2 = (\pi_{Yq})^2 - \pi_{YY}\,\pi_{qq}$ and is the determinant used to evaluate the familiar second-order sufficient conditions for a two-variable unconstrained maximization problem. If, as is often done, it is assumed that the second-order conditions are satisfied, then D_2 is positive. In the numerator, however, only π_{YY} can be shown to have determinate sign; the π_{ij} cross partials are not known a priori.[18] This means, therefore, that the sign in Equation (30) is ambiguous in the general case. In other words, theory cannot say beforehand how a typical water-using producer's demand for water will react, for example, to a more stringent turbidity concentration standard.

Additional insight into this result can be gained by examining the special case in which the price p_z at which Product z is sold is parametric to the producer (as would be the case if the market for z were perfectly competitive). Under such condition, p_z assumes a set value and therefore is not a function of z ; in other words, the producer faces a horizontal

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17/ Let Equation (26') be denoted in functional form as simply $\pi(q, Y, s, p_y, p_w)$. First-order maximization Equations (28) and (29 can then be written as

$$\pi_Y(q, Y, s, p_y) = 0 \quad \text{and} \quad \pi_q(q, Y, s, p_w) = 0.$$

Using double subscript notation for second partials, the comparative statics total differentials are:

$$\pi_{Yq}dq + \pi_{YY}dY = -\pi_{Ys}ds - dp_y$$

$$\pi_{qq}dq + \pi_{Yq}dY = -\pi_{qs}ds - dp_w$$

Now, divide through by ds , hold p_y and p_w fixed, and use Cramer's Rule to derive Equation (30) in the text.

18/ See Appendix B for some of the details on evaluating the π_{ij} terms.

demand curve (which means $p_z' = 0$ in Footnote 15 so that $MR_z = p_z$). Without detailing the algebraic derivation steps, the result of calculating, and substituting appropriately for, the π_{ij} terms is

$$D_2 \cdot \frac{\partial q}{\partial s}\Bigg|_{\substack{\text{para-}\\ \text{metric}\\ p_z}} = MR_z^2 \left[-Z_{YY}(q[Z_{qT} + sZ_{TT}] + Z_T) + qZ_T(Z_{Yq} + sZ_{YT})\right] \quad , \quad (31)$$

where the D_2 denominator from (30) has been cross-multiplied for easier presentation and, furthermore, is understood to be internally modified in whatever way necessary to reflect the "parametric p_z" assumption, although this does not change its being positive according to the second order maximization conditions. By recalling the definitional discussion of the Z( ) production function and its marginal productivity characteristics, it can be expected that $Z_{YY} < 0$, $Z_{TT} > 0$, and $Z_T < 0$, but signs for the Z_{ij} cross-partials are not known. More important, however, is the fact that when plausible sign combinations are postulated, a determinate sign for (31) is still not possible unless relative magnitudes are known. In short, even in the special (simplifying) case of parametric p_z , the sign of $\frac{\partial q}{\partial s}$ is still ambiguous.

Similar, therefore, to consumer water demand, theory is not able to predict unambiguously how producer derived demand for water will respond to changes in s. (The demand function may shift inward, outward, or may even pivot, as in <u>Figure II-2.</u>) The derivation of Equation (30), however, gives the technically correct methodology to use in a specific instance. This methodology is explicitly demonstrated in the following section.

B. ILLUSTRATIVE EXAMPLE

Paralleling the development of Chapter II's consumer demand example, it is possible to postulate a hypothetical production function for Product z which will play the role of $Z(q, Y, T)$ so that an explicit derived demand function for water can be deduced. The effect of a change in s can then be calculated. In addition, the corresponding marginal revenue function can be derived so that "marginal revenue = marginal cost" equilibrium can be examined.

Demand Function and Quality-Induced Shifts

Turning to the formulation itself, assume that the production process is characterized by this function:

$$z \equiv Z(q, Y, T) \equiv \sqrt{Yq^2/T} \quad . \tag{32}$$

Direct computation verifies that $Z_q, Z_Y > 0$; $Z_T < 0$; $Z_{YY} < 0$; and $Z_{TT} > 0$ showing that Function (32) exemplifies the marginal product characteristics cited in the previous Section.[19]

When (32) is inserted into Equation (26), profit π becomes an explicit function of Y, q, and T; ready to be maximized subject to the turbidity constraint of Equation (27). As noted, however, the latter relation can be substituted directly into the profit function, eliminating T as an explicit decision variable and thereby directly incorporating the constraint so as to allow an unconstrained optimization with respect to q and Y.

19/ The only condition cited earlier that is not met is $Z_{qq} < 0$ since here $Z_{qq} = 0$. Theoretically speaking, however, this does not preclude achieving an optimum, for marginal revenue product ("MRP_q" in Footnote 15) is still down-sloped, and that is the factor critical to optimization. Practically speaking, substitution of the turbidity constraint <u>does</u> make $Z_{qq} < 0$, but, more important, the complete model here does achieve a maximum since the second-order conditions are fulfilled (see Appendix B).

Assume now that the producer faces a linear downward-sloping demand curve for Product z, the equation for which is

$$p_z = -az + b \qquad ; \quad a, b > 0 \quad . \tag{33}$$

In other words, an example of the general case of non-parametric price p_z (and hence non-constant marginal revenue) is being considered. When (32) is substituted for z in (33), and also the turbidity constraint is incorporated, then p_z itself becomes a function of q and Y.

Implementing the steps of the two preceding paragraphs gives the following profit objective function:[20/]

$$\pi = -(aYq)/s + b\sqrt{Yq/s} - p_y Y - p_w q \quad , \tag{34}$$

for which the first-order maximization equations ($\pi_i = 0$, for $i = Y, q$) are:

$$\left.\begin{array}{l} -(aq)/s + b\sqrt{q} \Big/ 2\sqrt{sY} - p_y = 0 \\ -(aY)/s + b\sqrt{Y} \Big/ 2\sqrt{sq} - p_w = 0 \quad . \end{array}\right\} \tag{35}$$

Although non-linear in form, Equations (35) are solvable by algebraic manipulation for the optimal q and Y expressions which constitute the derived demand functions for them. Only the water demand function is of interest here (although the function for Y is entirely symmetric; merely reverse the positions of p_y and p_w):

$$q = (b\sqrt{sp_y}) \Big/ (2a\sqrt{p_w}) - (sp_y)/a \tag{36}$$

It is clear that q is inversely related to p_w (downward-sloping property), and the quality parameter s appears explicitly.

By direct differentiation,

$$\frac{\partial q}{\partial s} = [(b/4\sqrt{sp_w p_y}) - 1](p_y/a) \quad , \tag{37}$$

which means that the derived demand for water shifts outward, not at all, or inward as s falls, depending on the square-bracketed term in (37) being

20/ That is, the total revenue portion of π is $p_z \cdot z = (-az + b)z = (-az^2 + bz)$, whereupon (32) is substituted for z , and (27) is inserted for T to arrive at (34).

negative, zero, or positive. Concurring therefore with the theory result in Equation (30), the example function does not yield the same result for all possible values of input prices, the output demand function parameter b, and turbidity concentration s. Their relative magnitudes are critical for determining what the exact effects will be. Graphically, an appropriate representation would be a diagram similar to Figure II-2 where a pivoting function was depicted. Figure III-1 gives the essential features: namely, for relatively high values of p_w, the bracketed factor in (37) could be expected to be negative, giving $\frac{\partial q}{\partial s} < 0$ so that a ("unit") drop in s would cause q to rise. Just the opposite occurs at relatively low values of p_w.

Further elaboration on the conditions for derived demand shift is possible and, in addition, is instructive because it can suggest a practical means by which water planners may be able to deduce what shifts to expect. That is, the following analysis is quite specific to the example being considered here, but the notion of price elasticity of demand is a general (and familiar) concept with intuitive economic meaning.

Defined, therefore, as the percentage change in quantity demanded relative to a corresponding percentage change in price, the price elasticity of demand for Product z can be written in mathematical form as:

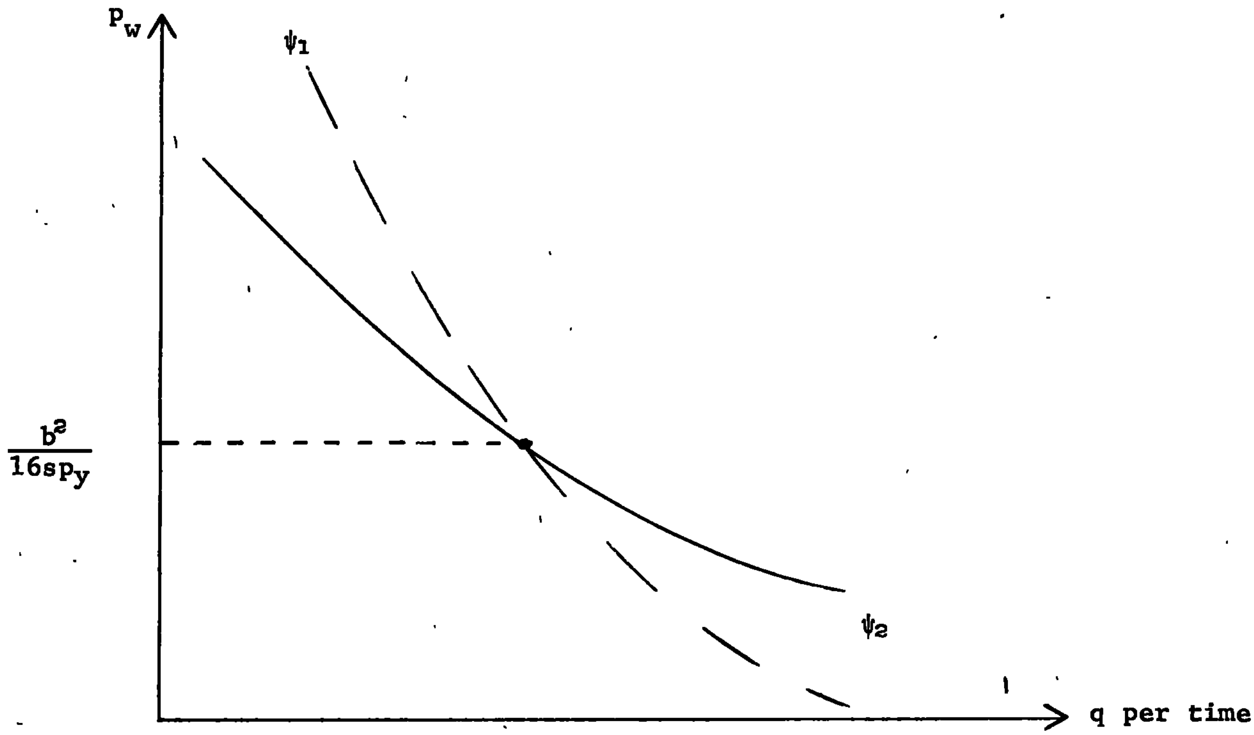
$$e_z = -(p_z)/z \cdot \frac{\partial z}{\partial p_z} \qquad (38)$$

where all components are calculated from the demand function for z, namely Equation (33). Computing the simple partial derivative and substituting for p_z gives

$$e_z = -1 + (b/az) \qquad (38')$$

FIGURE III-1

SKETCH OF PRODUCER'S DERIVED WATER DEMAND CURVE
FOR ILLUSTRATIVE EXAMPLE*/



ψ_1 is the derived demand function for more stringent standard (lower value of) s_1 as compared with s_2, to which ψ_2 corresponds.

*/ See Equation (36) of the text. The footnote to Figure II-2 regarding graphical representation of calculus results applies here as well.

It is now desired to find how e_z is related to the producer's optimal level of z, because when this is done, (since optimal z is a function of optimal q) the effect of s on q can be expressed in terms of e_z. In other words, Equation (37) could be rendered as a function of e_z.

Merely describing the steps, but not reporting derivational details, the solutions for q and Y (as given by Equation (36) and a similar one for Y) are substituted into Equation (32)'s production function with T eliminated, namely $z = \sqrt{Yq/s}$. Substituting the result of these steps for z in (38') gives $e_z = (b + 2\sqrt{})/(b - 2\sqrt{})$, where "$\sqrt{}$" $= \sqrt{sp_w p_y}$. Finally, some algebraic rearrangement of (37) gives the result that

$$\frac{\partial q}{\partial s} \gtreqless 0 \Longleftrightarrow e_z \lesseqgtr 1 + b/2\sqrt{} \qquad (37')$$

where, considering only the positive root, the last expression written is greater than one. From (37') it can be deduced (bottom inequalities) that, <u>if</u> q and s move opposite each other, <u>then</u> output elasticity must exceed 1. (That is, a negative partial derivative implies $e_z > 1$, but not the converse.) In economics jargon, a stricter turbidity concentration standard can be expected to cause an increase in demand for water input only if demand for Product z is elastic. Or (reversing the logic flow), if output demand is <u>not</u> elastic, then one would not expect the derived demand curve for water to shift outward with a fall in s. One intuitive explanation of this result would be that, although a higher water quality standard might seem to encourage the producer to use more water, the fact that he can reduce his output without decreasing his revenue from sales (this follows from $e_z \leq 1$) means that he actually has incentive <u>not</u> to increase his water use. Thus, as indicated in the illustrative example, knowing the price elasticity of demand for output can be useful information for helping ascertain how a change in quality factor will affect a producer's demand for water.

Attempts to derive a general relation between $\frac{\partial q}{\partial s}$ and e_z in Section III-A proved unsuccessful; it is likely that it does not exist. As was the case here, however, some kind of relation may be derivable for any particular instance; if so, it could constitute a pragmatic water planner's tool for helping assess the impact of quality considerations on water usage.

Marginal Revenue Function and Equilibrium Shifts

It was seen in Chapter II that if a "quantity-as-a-function-of-price" demand function formulation is rearranged to give the "inverse" demand relation of price as a function of quantity, then this facilitates derivation of a marginal revenue function. Accordingly, Equation (36) can be solved to get

$$p_w = (b^2 sp_y)/4a^2 \ (q + sp_y)^2 \quad ,$$

and then total expenditure (revenue received) on water, $p_w \cdot q$, can be expressed solely in terms of q by multiplying the numerator of this solution by q. Differentiation with respect to q then gives the marginal revenue function corresponding to the example producer's derived demand function as:

$$PMR = b^2 sp_y(sp_y - q)/4a^2 (sp_y + q)^3 \quad . \tag{39}$$

Marginal revenue shift in response to a change in s is, from an overview perspective, indeterminate, because

$$\frac{\partial PMR}{\partial s} = b^2 p_y [-3s(-) + (2sp_y - q)(+)]/4a^2 (+)^4 \quad , \tag{40}$$

where the symbols $(\pm)$ denote $(sp_y \pm q)$, respectively. Thus, here again one must know the exact values of the terms in order to know the sign of the derivative. Only in the case where q lies in the range $sp_y < q < 2sp_y$ can one note that unambiguously PMR will move with s, signifying that for that range of q values a more stringent turbidity standard would cause a drop (inward shift) in derived demand marginal revenue.

If one now assumes, for methodological presentation purposes, that Equation (36) is the derived demand curve for a group of water-using producers,[21] and Volume Two's CEM marginal cost function is again taken to represent the relevant treatment costs, then a ("partial") equilibrium analysis similar to that in Chapter II can be performed. Since it is virtually the same sort of analysis, reference should be made to the discussion leading up to Equations Equations (24) and (25) for operational details. If this is done, then it is sufficient simply to present the essential formulations and results.

Accordingly, Footnote 12 recalled the relevant marginal cost function MC. When equilibrium is characterized by MC = PMR (where PMR is given by Equation (39) above), then a polynomial in q of degree seven results, for which no analytic solution can be obtained. Again, however, the desired relation can be deduced by comparative statics analysis of that equation, yielding a result that is rather complex in form:[22]

$$\frac{\partial q}{\partial s} = \frac{(2spy - q)\ VR - 4a^3(\ +\ )^2\{2AR(\ +\ ) + 3p_y[\quad]\}}{4a^2(\ +\ )^3\ (3[\quad] + sA) - sV\{(\ -\ ) - q\}/4R} \tag{41}$$

where the short-hand symbols used are:

$(\pm)$ = as defined for Equation (40)

$V = 2b^2 p_y$

$R = \sqrt{q}$

$[\quad] = 2sAR + D$

21/ As was done in Chapter II, summing demand functions to obtain an aggregate demand function is certainly possible, but the form of (36) would not be preserved unless all producers had identical production functions and faced identical demand functions for their outputs. Hence the heroic simplifying assumption is made here for convenience since nothing is lost in demonstrating methodology.

22/ The turbidity concentration symbol s is used here, but its meaning is the same as s' in Chapter II.

A, D = marginal cost function parameters.

Equation (41) defies attempts to generalize its sign; there is no question but that it can be +, - or 0, and, as could be expected, the value of q is critical to determining what the sign will actually be. Thus, just as was the case with the consumer demand example, the method for measuring effects of changes in s on a water-using producer's demand for water is straightforward and logical, but the results cannot be predicted ahead of time from "symbolic-only" functional forms; specific parameter values must be known.

IV. AGGREGATE JOINT DEMAND

In Chapter II, the combination of consumer demand functions by "adding" similarly-derived functions was shown to yield what can be interpreted as an aggregate consumer water demand function; comment in this same regard for producer derived demand functions was made in Chapter III. It is now possible to demonstrate how the two types of demand can be combined into an "overall" aggregate function. Intuitively, of course, one would expect this to be the case so long as both functions are defined in the same units. In all the discussions developed here, the functions express water quantity demanded as a function of its price per unit (gallons, say), so the requisite consistency is immediately established. In addition, all the example functions contain a quality parameter which measures exactly the same thing.

The question of comparable units of measurement thus settled, derivation of the aggregate joint demand function is relatively straightforward. In the following section, the joint demand function corresponding to Chapters II and III's example functions is given, and then the corresponding marginal revenue function is derived. From this result, the way toward equilibrium is pointed, but, unlike what was done in the preceding chapters, no comparative statics analysis will be performed due to the complexity of the functions involved. Since, however, the technique that would be used is precisely the

the same as that used before, "stopping short" really does not constitute an omission of information from a methodological standpoint.

A. CONSUMER AND PRODUCER DEMANDS COMBINED

As noted, the hypothetical consumer demand function of Equation (21) and the example derived demand function in Equation (36) are immediately additive because measurement units are the same even though the demands emanate from different sources. For notation clarity, let left-side value q be denoted q_P in (36), and take q_C from (21); the subscripts P and C signify producer and consumer quantities demanded, respectively. Then the example joint demand function can be written (letting $\overline{q}$ now denote aggregate quantity demanded):

$$\overline{q} = q_C + q_P = \underset{\text{Equation (21)}}{\text{Right Side,}} + \underset{\text{Equation (36)}}{\text{Right Side,}}$$

or, after algebraic simplification,

$$\overline{q} = (M'/p_w) + b\sqrt{sp_y}\Big/2a\sqrt{p_w} - (p_y + k')s \qquad (42)$$

where s is both s' and s from the component functions. It is hardly necessary to point out, therefore, that the joint demand function is downward sloping with respect to water price p_w, and it incorporates the turbidity concentration water quality parameter explicitly.

Continuing now toward the joint marginal revenue function,[23] the inverse demand function is obtained, as before, by solving the demand function

[23] It should be noted that the joint marginal revenue function cannot be derived by simply summing individual functions because mathematically that procedure would constitute "vertical" addition, whereas "horizontal" addition is required. One must first derive the joint demand curve and then obtain the marginal revenue function that corresponds to it.

for p_w as a function of quantity. In the current case, a quadratic equation results from which two roots can be obtained analytically. With p_w a function of $\bar{q}$, total revenue thus also is in terms of $\bar{q}$, and marginal revenue is found by differentiating with respect to $\bar{q}$. Let

$$\Lambda = b\sqrt{sp_y}/2a \quad , \quad \Omega = (\bar{q} + sp_y + sk') \quad , \text{ and } \quad \Gamma = \sqrt{\Lambda^2 - 4M'\Omega}$$

for notational efficiency (observe that Λ is not a function of $\bar{q}$, but Ω is). Then, omitting all derivational steps, the rather complex expression for marginal revenue is

$$\overline{MR} = \{2\,\Omega^2\,[\Lambda^2 \pm \Gamma^{-1}\,(\Lambda^3 - 4M'\Omega\Lambda - 2M'\Lambda\bar{q}) - 2M'(\Omega + \bar{q})]$$

$$- 4\Omega\Lambda\bar{q}\,[\Lambda \pm \Gamma - 2M'\Omega/\Lambda]\}/4\Omega^4 .$$

Equilibrium analysis would now involve equating $\overline{MR}$ to marginal cost and deducing $\frac{\partial\bar{q}}{\partial s}$, as demonstrated before. Nothing more than a cursory glance at Equation (43), however, is needed to indicate that such analysis would be very involved algebraically. It is virtually certain that the sign of the derivative will be ambiguous for the general-value case; hence, numerical evaluation for each specific situation would be required. Since the methodology is just like that used before, little could be gained by deriving $\frac{\partial\bar{q}}{\partial s}$ here. In addition, one would expect a graphical representation to show shifting/ pivoting characteristics, just as with the previous figures drawn.

B. EMPIRICAL IMPLEMENTATION--BRIEF COMMENT

Although the chief purpose of this volume has been to demonstrate the conceptual rationale for water demand functions which "pay attention" to water quality factors, it is well-recognized that this is not an end in itself. Water planners would obviously like to feel that there is empirical applicability to the analyses/results that have been presented. This is indeed the case.

In Chapter II, the virtual impossibility of deducing utility functions was cited, leading to the ultimate suggestion that an aggregate (of consumers) analysis would be more feasible. Econometric regression technique for obtaining demand function estimates was mentioned as a viable tool.

Somewhat the same argument, from a practical standpoint, could be made with respect to derived demand. The major distinction, however, is the fact that, unlike utility functions, production functions (the Z( ) function of Section III-A) are empirically determinate, either from engineering sources, or, again, by estimation. It is thus possible to perceive quantifying a demand function like Equation (30) or (36). On the other hand, the relatively simple example formulations used here showed that, mathematically speaking, "complete" derivations (i.e., obtaining demand functions from the optimization model itself) may be all but impossible.

For this reason, a major purpose has been served if, as was suggested in the "supply side" analyses of Volumes One and Two, the work done here can indeed be considered documentation of the theoretical basis for perceiving demand functions that account for quality in an explicit way. Thus, with the background developed here, one could legitimately hypothesize an empirically estimable aggregate joint water demand function playing the role of Equation (42) and proceed, with data, to deduce a function directly by statistical means. The statistical analysis itself can indicate if a "good" form has been chosen. From this point (depending on complexity of functional form), partial equilibrium analysis as shown can then be conducted.

As a concluding remark on the entire study, it is appropriate to reiterate that the central purpose of the project has been met. It has been shown that the notion of "water supply analysis" incorporating explicit quality factors rests on solid economic theory grounds. Thus, it makes sense to conceive of "water supply" changes in response to changes in quality parameters.

Our analyses, particularly in Volume Two, have, moreover, been stepwise procedures that could be empirically implementable, given data availability and proven numerical analysis techniques. In a sense, then, this would constitute a challenge for future work and research. What has been accomplished here is to "break ground," so to speak. Fruitful effort should not be directed toward generalizing the results given here (e.g., multiple impairments) and empirical implementation. Hopefully, this will indeed come about.

V. REFERENCE LIST/BIBLIOGRAPHY

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APPENDIX A

CONSUMER DEMAND MATHEMATICAL DETAILS

This appendix gives "main point" derivation details for the consumer demand results presented in Chapter II. In doing so, critical steps are outlined for three topics, but intermediate stage algebraic manipulations are omitted.

Derivation of Conceptual $\frac{\partial q}{\partial s}$ for Equation (12)

Equation (11) of the text gave the first-order utility-maximization condition, it being simply the first derivative of the utility function (with respect to q, since G and T were eliminated by substituting from the budget and quality constraints) equated to zero. Because each of the partial derivatives U_j is a function of q, G and T, Equation (11) is too. In the comparative statics analysis leading to Equation (12), therefore, there is contribution from each variable.

More specifically, denote the first partial as $\varkappa_j = U_j(q,G,T)$ from which follows the total differential $d\varkappa_j = U_{j\,q}d_q + U_{j\,G}dG + U_{j\,T}dT$. (Compare this with Footnote 6 of the text.) Upon substituting for dG and dT computed from Constraints (2') and (3'), one obtains

$$d\varkappa_j = [U_{j\,q} - (p_w/p_g)\,U_{j\,G} + sU_{j\,T}]\,dq + qU_{j\,T}ds \quad . \tag{A-1}$$

When the differential of Equation (11) is now computed, each of the three dU_j (j = q, G, T) factors that will appear must be replaced by an (A-1) expression. Schematically, this differential is

$$-(p_W/p_g)\, d\varkappa_q + d\varkappa_G + s d\varkappa_T + U_T ds = 0 \quad . \tag{A-2}$$

Substituting for each $d\varkappa_j$ factor renders (A-2) solely in terms of the differentials dq and ds , whereupon like terms can be collected so that division through by ds will give Equation (12), where the partial derivative interpretation there reflects the fact that prices and money were held constant.

Derivation of Hypothetical Example Consumer Demand Function

Equation (13') gave the illustrative example utility function with budget and turbidity constraints already incorporated to make utility a function of only q. By chain rule differentiation, the derivative of U with respect to q is

$$\frac{dU}{dq} = \alpha[\quad]^{\alpha-1} \cdot \left(\frac{1}{s} - \frac{p_W}{M-p_W q}\right) \tag{A-3}$$

where [ ] is the square-bracketed factor in (13'). Setting (A-3) equal to zero is possible if and only if, the parenthesized portion is zero. Equating that part to zero and solving for q gives the optimal value of q , namely the demand function of Equation (14).

Second-Order Conditions for Hypothetical Example

The second-order sufficient condition for establishing Equation (14) as the true utility-maximizing solution for the illustrative example is that the second total derivative of U with respect to q should be negative, when evaluated at the solution given by (14). This is a relatively straightforward calculation; one merely differentiates (A-3) with respect to q, and then sets equal to zero the parenthesized factor in (A-3) wherever it appears. The result of this exercise is

$$\frac{d^2U}{dq^2} = -\alpha[\quad]^{\alpha-1}\, p_w^2 \,/(M - p_w q)^2 \qquad , \qquad \text{(A-4)}$$

which must be negative since the exponentiated square-bracketed term, defined in (A-3), is positive by the previously noted positive marginal utility properties. The solution in text Equation (14) is thus the optimal solution and hence is indeed the consumer's demand function for water.

APPENDIX B

PRODUCER DERIVED DEMAND MATHEMATICAL DETAILS

The purpose here is to present derivation highlights of some of the results reported in Chapter III.

Background for Conceptual Results of Section III-A

It was seen that the conceptual form for III-A's production function was given as $z = Z(q,Y,T)$. Similar to the analysis in Appendix A, the total differential of this function can be written, after incorporating the turbidity constraint $T = sq$, as

$$dz = (Z_q + sZ_T)dq + (Z_Y)dY + (qZ_T)ds \qquad \text{(B-1)}$$

For the profit-maximization analysis, the partial derivatives of the profit function of Equation (26') entail computing by chain rule the partials of total revenue TR as $\frac{\partial TR}{\partial k} = \frac{dTR}{dz} \cdot \frac{\partial z}{\partial k} = MR_z \cdot \frac{\partial z}{\partial k}$ $(k = q,Y)$ where MR_z is marginal revenue as defined in the text and $\frac{\partial z}{\partial k}$ is the parenthesized coefficient of dq or dY from (B-1) for k = q or Y, respectively. (Refer to Equations (28), (29) and Footnote 15 of the text to see directly where this result is used.)

Details on π_{ij} Terms in the Maximization Model

Unconstrained maximization of the schematic form profit objective function of Equation (26') led to first-order Equations (28) and (29), both of which contain partial derivative ("marginal product") factors Z_i, each of these being a function of the three variables q, Y, and T in the general case. Analogous to Appendix A's results on differentiating utility function partial derivatives, the total differential of Z_i involves differentials of all three variables. When the turbidity constraint is then incorporated to give $dT = sdq + qds$, the differential of Z_i becomes

$$dZ_i = (Z_{i\,q} + sZ_{i\,T})dq + (Z_{i\,Y})dY + (qZ_{i\,T})ds \tag{B-2}$$

from which the partials of Z_i with respect to q and Y can be read as simply the coefficients of dq and dY, respectively.

To demonstrate the use of this result, recall that Equation (28) shows that $\pi_Y = MR_z \cdot Z_Y - p_Y$. The total differential is thus (assume p_Y constant)

$$\begin{aligned} d\pi_Y &= MR_z dZ_Y + Z_Y dMR_z \\ &= MR_z(dZ_Y) + Z_Y MR_z'(dz) \end{aligned} \tag{B-3}$$

where MR_z' is the derivative of MR_z with respect to z. The parenthesized differentials are taken directly from (B-2), with i = Y, and (B-1). Once these substitutions are incorporated and terms collected, cross-partials are calculable. For example, if one divides (B-3) through by dq and then holds Y and s fixed, the result is

$$\pi_{Yq} = MR_z(Z_{Yq} + sZ_{YT}) + Z_Y MR_z'(Z_q + sZ_T) \tag{B-4}$$

where the parenthesized factors will be recognized as the coefficients of dq in (B-2) and (B-1).

Similar computations give expressions for the other two second partials and thus enable evaluating the general form comparative statics result in Equation (30), one special case of which is shown in Equation (31) written out in terms of expressions like (B-4).

Derived Demand Example: Solutions

Transpose the terms with minus signs in Equations (35) to the right-hand sides, multiply through by s and then "divide" one equation by the other (i.e., divide left side by left side, and similarly for right sides). Some algebraic simplification gives the result $p_w q = p_y Y$ which can be solved for Y as a function of q. Substituting back into either Equation (35) will then give the water derived demand function of Equation (36); a similar process gives a symmetric solution for Y. Substituting both solutions into $z = \sqrt{Yq/s}$ gives optimal $z = (b/2 - \sqrt{sp_w p_y})/a$. Note that this is used to derive the elasticity relation in Equation (37').

Derived Demand Example: Second-Order Conditions

Two-variable unconstrained optimization theory notes that _if_ specified combinations of second partials π_{ij} have certain signs, when evaluated at a "candidate" optimum, _then_ the candidate solution is unambiguously a maximum. For the text example, either applying general formulae like (B-4) or simply differentiating directly the left sides of Equations (35) gives:

$$\left.\begin{aligned} \pi_{YY} &= -b\left(\sqrt{q/sY^3}\right)/4 \\ \pi_{qq} &= -b\left(\sqrt{Y/sq^3}\right)/4 \end{aligned}\right\} \quad \text{positive root implies unambiguously negative}$$

$$\pi_{Yq} = -a/s + b/4\sqrt{sYq}$$

The determinant D_2 defined at Equation (30) then is

$$D_2 = (a/s^2)\,[-a + b\left(\sqrt{s/Yq}\right)/2]$$

which is unambiguously positive when the Y and q solutions are inserted. The signs cited are precisely those needed to satisfy the second-order conditions. Hence we have verification of a true maximum (and therefore true derived demand functions).

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13. ABSTRACT

The purpose of this study was to examine the feasibility of embedding quality parameters explicitly in water supply analyses. A microeconomics approach using constrained optimization was employed, and the study was written in three volumes. Volume One presents water supply function conceptual theory and methodology by means of classical economics model (CEM) derivation of a treatment plant's marginal cost function with a water quality-related parameter incorporated explicitly. Such formulation enables computing the marginal cost of removing incremental amounts of an impairment. A linear programming model (LPM) then is developed to demonstrate how it is possible to depict internal behavior by a water-using producer who can alter his "effective" supply by upgrading lower quality water to higher acceptable quality. Volume Two is the empirical implementation of Volume One. For the CEM, actual data were used to help estimate a sample cost function, while dummy data enabled demonstrating the sensitivity analysis capabilities of the LPM. Both models permit estimating costs of removing water contaminants. Volume Three gives the conceptual justification for water demand functions (for consumers and producers) with explicit quality parameters incorporated. Hypothetical examples are given, and, by combining these with Volume Two's CEM, the sensitivity of "marginal revenue=marginal cost"equlibrium to quality parameter shifts is examined by means of comparative statics analysis. Throughout the study, a measure of turbidity concentration is used as the illustrative quality parameter, but the conceptual feasibility of alternative forms is discussed. Volume One, for instance, presents a literature survey of proposed measurable water quality indexes.

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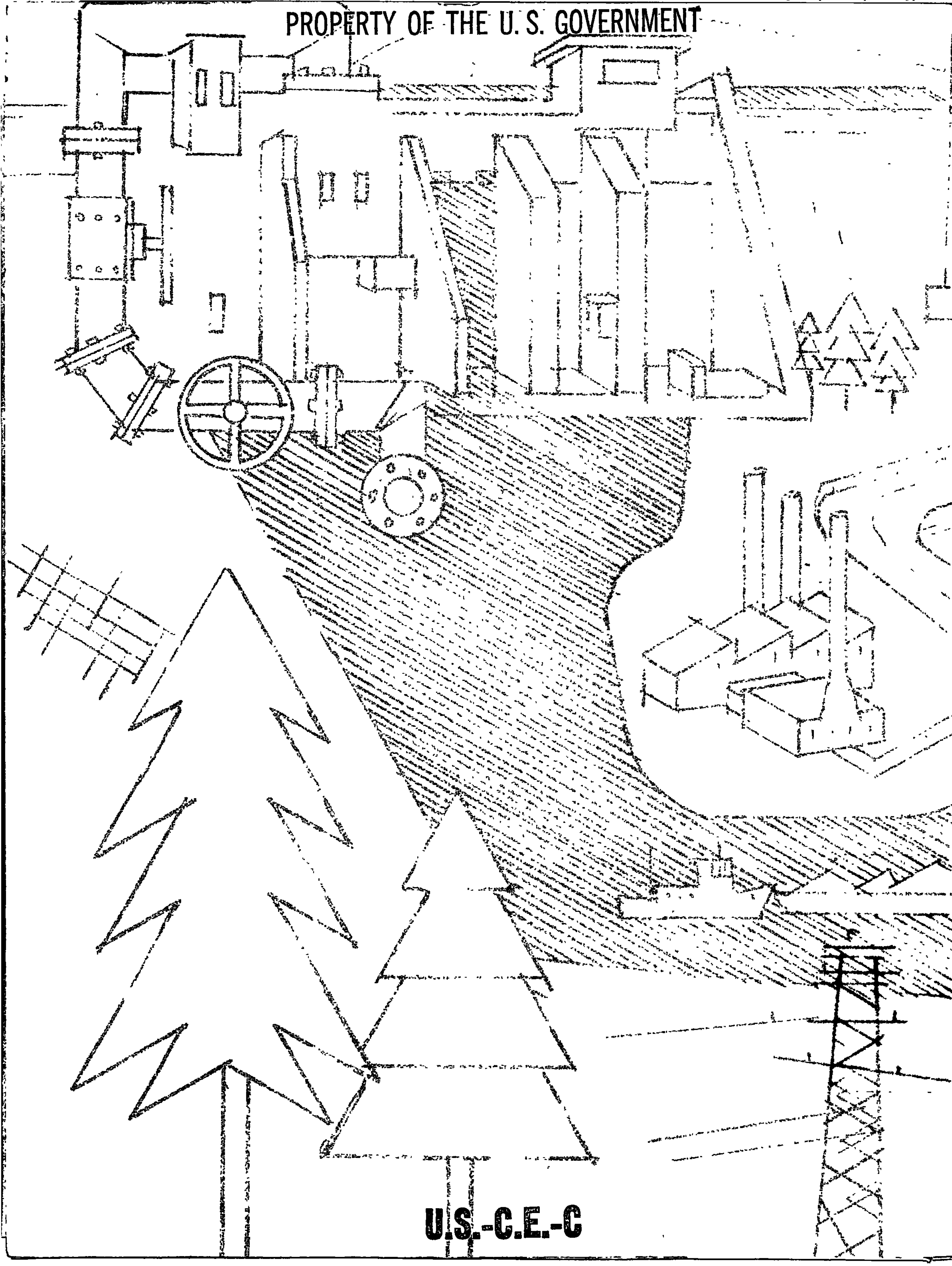
Comparative Statics Equilibrium Analysis						
Constrained Profit--Maximization						
Constrained Utility--Maximization						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